

Partitions!

Partitions are ways to add up to a certain number. You can use a number more than once, and the order doesn't matter. We write partitions from greatest to least. For example, here are three partitions of the number 8:

$$7+1$$

$$4+2+2$$

$$3+3+2$$

$1 + 7$ is not considered a different partition from $7 + 1$.

As another example, here are all the partitions of the number 4:

$$4$$

$$3+1$$

$$2+2$$

$$2+1+1$$

$$1+1+1+1$$

There are five partitions of the number four.

1) Find all the partitions of the numbers from 1 to 8. List them in a neat orderly way, and count how many partitions each number has.

2) What do you notice so far?

3) Copy and continue a function table like the one below showing the number of partitions for each number. Can you find any patterns? Can you make a prediction?

number	number of partitions
1	1
2	2
3	3
4	5
5	7

4) Can you go further than 8 in listing all partitions? Warning: the number of partitions grows quickly! For example, there are 627 partitions for the number 20.

5) A Ferrers Diagram is a way to show a partition as a drawing of dots in rows. For example, here is a Ferrers diagram showing one partition of 9:

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4 ****
3 ***
1 *
1 *
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Illustrate some of your partitions as Ferrers diagrams.

6) How many partitions of 7 have 4 as the biggest number in the partition? How many partitions of 7 have exactly 4 addends? Is it a coincidence that these numbers are equal? Will this always happen? Can you find a way to understand why this happens using Ferrers diagrams?

7) The great mathematician Leonhard Euler proved that, for any number, the number of partitions using odd numbers only is always equal to the number of partitions with distinct addends (no repeats). For example, three has two partitions with odd numbers only: 3, and $1 + 1 + 1$; and two partitions with distinct numbers: 3, and $2 + 1$. Test Euler's theorem! Do you agree?

8) The great mathematician Ramanujan proved that the number of partitions for numbers ending in 4 or 9 is always a multiple of five. Well, four has five partitions, so that checks out. If you're feeling brave, test Ramanujan's theorem for nine and fourteen!

9) Mathematicians are still learning new things about partitions, and how to know how many partitions each number will have. Do some research and see if you can learn more about partitions. Can you find out how many partitions there are of 100? Of 1,000??!!

10) Have fun!