PFS 34-3 11/3/20 ____________________

3n + 1

This is a famous unsolved problem. Here's how it works. Start with any counting number. If it's odd, multiply it by three and then add one. If it's even, divide it by two. Then iterate—repeat the same process over again with that new number. Here's what happens if we start at 5:

$5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 2 \rightarrow 1...$

As you can see, after five steps the number 5 reaches the number 1, and then we're in a cycle.

1) Try this iterative process with a few counting numbers between 1 and 20. Write down the path for each number, as shown above. Then write a few sentences about what you notice and anything you wonder about.

2) Show what happens to every number between 1 and 20, and for each number indicate the length of the path till 1 is reached. This is easier than you may think! Look at the example above. It doesn't just show what happens if you start with 5; it also shows what happens if you start with 16 or 8 or 4...

So what's the unsolved problem? No one knows if every counting number will lead, eventually, to the 4, 2, 1 cycle. Mathematicians have tried this with every number up to a trillion trillion, and they all go into that cycle. Mathematicians *conjecture*, or believe, that every number will go into that cycle, but no one has proved that they all will, and no one has found a number that doesn't!

3) If we ever find a number that doesn't go into the 4, 2, 1 cycle, what might its path look like?

4) Try the same procedure, but this time with the rule 3n - 1. Do you notice some differences?

5) Try the original 3n + 1 problem, but starting with any integer—in other words, you can start with zero or negative integers. What differences do you notice?

6) Try the original 3n + 1 problem with every number between 21 and 30. You may need a little patience! Write down what you notice.

7) Based on the work you've done so far, make a function chart showing numbers and their path length to 1. (Mathematicians call that the “total stopping time” of n.) For example, the total stopping time of 7 is 16. What do you notice?

8) Graph some of these paths on a coordinate plane. Do you see why these are sometimes called “hailstone sequences”?

9) The 3n + 1 problem is also called the Collatz Conjecture. Can you do some research on it?

Have fun!