

Regions in a Circle

Let's put points on a circle and connect each point to every other point with straight lines. The lines are like walls that make rooms or regions. How many regions will we get?

One point leaves one big region. Two points create two regions. Three points create four regions. What will happen with four points? With more? You can choose to have your points evenly spaced around the circle or not.

1) Make some diagrams with circles, points on the circles, and straight line segments connecting each point on the circle to every other point on the circle. Count the number of regions you get for each number of points on the circle. With six or more points, be sure to use LARGE full-page diagrams. Use a protractor to draw neat circles and to measure arcs. Use a ruler for your straight lines. Work neatly please!

2) Make a chart keeping track of the points, lines from each point, total lines, and total regions. How far can you get? Do you see any patterns? Are you working with points evenly spaced or not? Try making a chart for both kinds of point distribution. Is there a difference? What is it? Make sure to check your chart with a friend.

3) Can you find a rule or formula to help you understand this function?

4) Does it matter if the points are equally spaced around the circle?

5) Can you prove or explain any formula you find? Hint: it's easier to find a formula if the points are not evenly spaced around the circle and you never have three (or more) lines meeting in one point. (I said "easier." I did not say "easy"!!)

6) Can you find out how many regions 10 points will lead to? 20 points?

7) Can you create a work of art using one of your diagrams?

Hints for the ambitious: This problem is related to the rule for the number of diagonals in a polygon with n sides. Can you find that formula? There is also a formula that relates all the points, segments and regions (also known as vertices, edges and faces) in any network. Can you find that formula? (This second one is often called Euler's formula, but be careful if you research it. Euler lived a long time and discovered many laws and formulas!)

8) Have fun!