

11/5/18

Number Necklaces

You have a whole lot of beads numbered 0 to 9. You may choose any two numbers to start your necklace (including choosing two of the same number). Then you add them to see which number to put on your necklace next. If the sum is a number bigger than 9, you use its ones digit to get your next bead.

Here's an example. Begin with 2-2. So the next will be 4 ($2+2$), then 6 ($2+4$). But then $4+6=10$, a two-digit number. So we use the 0 from 10 to get our next number, 0. The beginning of this necklace will look like this: 2-2-4-6-0-6-6-2. A necklace isn't complete when you come full circle and get the first two numbers you started with.

- 1) Choose two numbers and make a necklace. Keep checking your work with a fellow student or an adult. When you get the same two numbers again, in the same order, snip them off. Your necklace is now complete. How many beads are in your necklace?
- 2) How many starting pairs of numbers can be made from the numbers 0 to 9?
- 3) Can you find a complete solution to this problem, that is, find all the possible necklaces? What are the lengths of your necklaces? What do you notice about them?
- 4) Which two starting numbers make the longest necklace? The shortest? Compare with a friend.
- 5) Must a necklace cycle around to the same two numbers in the same order? Why or why not? Could a necklace go on forever without coming back to its starting numbers?
- 6) Try writing your longest necklace in rows of ten, left to right. What do you notice?
- 7) Try making number necklaces in other bases. For example, the prime factors of ten are two and five. Compare the Base Ten necklaces to those with Base Two and Base Five. What do you notice?
- 8) Can you make up a similar problem?
- 9) Have fun!