RATS!

RATS stands for Reverse Add Then Sort. It's an operation made up by John Conway, a professor of math at Princeton University. The fun part is seeing what happens to different numbers as you do this operation over and over. (In math language, when we *iterate*.) Here's an example.

Let's start with 157. First we reverse it to get 751. Then we add:

```
      157
+     751
      908
```

Then we sort the digits in 908 smallest first. We get 089. If there are zeros in front we discard them. So we end up with 89. Then we do it all again with 89:

```
       89
+      98
      187
```

We sort 187 to 178. So far the three numbers in our path, or sequence, or trajectory, are 157, 89, 178.

1) Try RATS starting with 9. Work neatly, and check your work with someone! What happens as you keep going? Write up what you notice.

2) This time start with 3,999. What happens?

3) Now use 2,079 as your seed. Do at least 13 iterations. What happens?

4) This time start with a nice small number, 3. Keep going! What do you notice?

5) Ready for a challenge? Try RATS starting with 1. Any surprises?

6) Try RATS starting with numbers you choose. What happens? What do you notice about the sequences we get starting with different numbers? Do different numbers end up with the same destiny after a while? How many different destinies did you discover?

Square and Chop!

Here is another fun iterative procedure. Unlike RATS, this one has only 100 possible seeds, so it's possible for a students (or better yet, a team of students) to understand and map this whole universe. Start with any number from 0 to 99. Square you number. (That is, multiply it by itself.) Then divide by 100 and take the remainder. (This is the same as saying, chop the number between the tens and hundreds columns, and take the two digits on the right.) Repeat!

For example, start with 84. $84^2 = 7056$. 7056/100 leaves remainder 56. So now we repeat with 56. $56^2 = 3136$. So 36 is our next number. And we keep going! Can you work in a group and find out what happens with every possible seed? Try to make a well organized diagram to show all your work. If you wish, explore what will happen if you divide by a different number, such as 10, 11, or 12. (With 12, for example, your starting numbers will be from 0 to 11.)

In Square and Chop we can be sure every number will eventurally go into a cycle or reach a fixed points. Why is that? Is the same thing true in RATS?

Can you invent an iterative procedure of your own?

Have fun!