

Pancake Problems

Phyllis is a waitress at a diner with an unusual problem. Bob, the short order cook, makes the best pancakes in town and the diner has gained a reputation as being a “must-do.” The diner is so popular, in fact, it has even been featured on Diners, Drive-Ins, and Dives. There is just one issue with Bob’s pancakes: they are all different sizes and Bob doesn’t really focus on serving the pancakes as much as he does making them. Phyllis knows that customers want their pancakes to look as good as they taste, and that means the biggest on the bottom, the smallest on top, and everything in order in between. When Bob sends Phyllis a stack of pancakes she must first use a spatula to flip the pancakes until she has them in the right order, biggest to smallest, bottom to top. Phyllis cannot touch the pancakes with her hands, she cannot take some off the plate, she can only stick a spatula underneath a portion of the stack and flip. Sometimes it only requires one flip, sometimes two or three, and sometimes none at all!

Recently, Bob has added a new jumbo stack to the menu, which features five pancakes instead of three and Phyllis is worried that this will make her job much harder. Can you figure out how this problem works, how to solve a mixed up jumbo stack, and put Phyllis’ mind at ease in the process?

- 1) Cut out five paper pancakes of different sizes. Number them 1-5 with 1 being the smallest and 5 the largest. Stack them in this order from bottom to top: 1,4,3,2,5. Can you rearrange the pancakes to the correct order using the same technique as Phyllis?
- 2) Phyllis says she can solve any short stack (three pancakes) in three moves or less. Is there a way to prove this? Can you make a chart showing the different short stack possibilities and solutions?
- 3) Phyllis would feel better if she knew the maximum number of flips needed for the jumbo stack (five pancakes). Can you determine the maximum number of flips needed to solve a jumbo stack?
- 4) Is there a pattern to be discovered? Does the number of pancakes directly impact the maximum number of flips? Make a chart showing the numbers of pancakes in relation to the maximum number of flips. Notice any patterns?
- 5) Bob is thinking of adding a Super Jumbo Stack (six pancakes). Make a prediction based on the pattern you’ve noticed and then test to see if you are correct. Phyllis is worried that adding another pancake might make her job much harder. Is she correct?
- 6) This problem, often referred to as “Pancake Numbers,” is unsolved. Mathematicians have only been able to prove solutions up to a 19-stack. Do some research and see if

you might want to take a crack at solving the Mega Jumbo Giant Super Duper Stack (20 pancakes).

Phyllis has become fed up with Bob and quit her job. She now works for Bubba (Bob's cousin) and has discovered that Bob's pancake problem is a family trait. Worse than this, Bubba is not nearly as good a short-order cook as Bob and always burns one side of the pancake. So now, Phyllis not only has to reorder the pancakes, she has to make sure that the burnt sides are facing down (perhaps not the best display of integrity). Phyllis is working hard to find a way to get her old job back, but in the meanwhile, let's see if we can help her out a bit.

- 7) Color one side of your paper pancakes to indicate the burnt side. Using your new stack of Bubba's cakes, try to solve a stack ordered from bottom to top: 1,4,3,2,5, with all of the burnt sides face up at the start. How many steps did it take?
- 8) Can you determine how many possible short stacks (three pancakes) are possible now? What is the maximum number of flips needed to solve any short stack?
- 9) Have fun and maybe make some pancakes at home (with parent's help and approval, of course) to test this problem out in real time! If you do, consider making a short video to show in class.

Special thanks to Numberphile for the inspiration for this problem and to *The Simpsons* writer David X. Cohen for creating the burnt pancake variation and proving that math and comedy DO go together!