PFS 30-7 02/20/17 _____

Never Underestimate the Power of the Fours

Numberphile (our favorite YouTube channel) recently posted a video showing that by using four 4's and a variety of mathematical operations, one can produce any number, up to infinity. How is this possible and can we do this without a degree in mathematics? Perhaps if we start small, use what we know, and look for patterns, we might be able to figure this out (or at least make a dent in infinity).

1) Let's start with something a bit simpler. Using four 1's and the four basic mathematical operations (addition, subtraction, multiplication, and division), how many numbers can you produce. For example: 1 + 1 + 1 +1 = 4. What is the largest number you can create?

2) Why don't we try this again with the number 2? Will four 2's allow you to make more numbers? How many more?

3) Having investigated four 1's and four 2's, what predictions can you make about working with four 3's? Make a prediction for both how many numbers you can produce and the largest number you can create. Check to see if your prediction was accurate by finding all of the possible numbers able to be created with four 3's and the four basic operations.

4) Having learned the basics on 1, 2, and 3, let's try using four 4's. Remember, at this point we're only using addition, subtraction, multiplication, and division. How many numbers can you create?

5) Rather than moving on to 5's, let's expand what we can do with 4's. What if we open up some more ways to work with the numbers? Let's use grouping to help structure the order of operations. For example: (4+4) X (4 + 4) = 64.

6) What if we allow for concatenation? What is concatenation? Concatenation is the act of putting two or more things together in order to produce something new. For example putting two 1's together to make an 11. In that way we could have 11 + 1 X 1 = 12. Revisit the four 1's, four 2's, and/or four 3's and see how many more numbers you can create with this new operation.

7) Now that we're able to make larger number like 44 with concatenation, let's introduce decimals so that we can make smaller numbers like 4.4 or .44. Note that 0.4 can be written as .4 and is valid. For example: 44/.44 = 100.

So far we've used pretty basic math concepts. Before moving on to the remaining problems, make sure you've fully explored #'s 1-7. If you feel you're ready to explore this problem more deeply, flip the page.

If you want to fully explore this problem be prepared to do a bit of research on new concepts and don't be afraid to ask John or your math teacher for help if there's something new here that you don't understand.

8) Do you know what an exclamation point means when placed after a number (4!)? It is NOT that the number is just really excited, but rather a mathematical function called factorial. In mathematics, the factorial of a non-negative integer n, denoted by n!, is the product of all positive integers less than or equal to n. Confused? Think of it this way, when you use factorial, you multiply a number by all of the positive whole numbers less than itself. For example 4! = 4 X 3 X 2 X 1. Try using factorial to find even more numbers made by four 4's.

9) There are a few others functions we could add to the mix. Exponenents and square roots will help you get even further, but what about using percentages such as (4/4)% (meaning 4/4 as a percentage), or other such tricky uses of mathematic procedures?

10) Want to head towards infinity? In order to get to REALLY big numbers, we're going to need to learn about logarithm. Using Log base 4, naturally, we can head towards infinity. If you're interested in learning about this function, do a bit of research and try using it to create a few numbers using the four 4's. The Numberphile video that inspired this problem can be found on Youtube by searching "numberphile fours." Be sure to get a parent or teacher's permission before heading to that video.

11) Have fun! How can you have fun with this problem? Find ways of getting to the same number using different methods. Figure out creative uses for mathematic principles we've learned in POW, such as triangular numbers or combinatrics. Work with a partner to see how many numbers you can generate in a worktime, WITHOUT talking to each other. Math students of all levels can have fun exploring the power of the fours.