Fractals!

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." – Benoit Mandelbrot

In this POW we're going to learn about two fractals. It's a great opportunity for some beautiful art work, but also a chance to learn about a relatively new part of math.

1) The Sierpinski Gasket

The Polish mathematician Waclaw Sierpinski first discovered the Sierpinski gasket in 1915. Start with an equilateral triangle. (Edges of 16 or 32 cm might work well.) Mark the midpoint of each edge. Connect the midpoints and "remove" the middle triangle by coloring it. Now repeat the same process with the three smaller equilateral triangles, and so on. Try using different colors at each level. How far can you go? What can you learn about the Sierpinski Gasket?

2) The Koch Snowflake

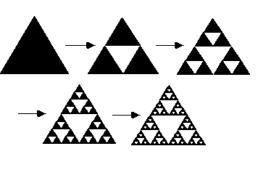
The Swedish mathematician Helge von Koch first described the Koch snowflake in 1904. We start with an equilateral triangle, and at each step we add equilateral triangles facing out to the middle third of every side. Here are the first four steps.

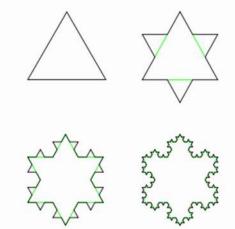
The Koch snowflake is what we get if we iterate forever. Make a neat drawing showing the first few iterations. You might want to start with a triangle 9, 18 or 27 cm long. Can you make your snowflake beautiful? If we continue forever, do you think the perimeter be infinite? Will the area be infinite?

3) Sides in the Snowflake

Make a function chart showing the number of sides for each level of the Koch Snowflake. Call the first equilateral triangle level zero. You chart will start like this:

Level:	0	1
Sides:	3	12





4) The Perimeter of the Snowflake

Now try to determine the perimeter for each level. Let's say you start with a triangle 27 cm on each side. The perimeter function will start like this:

Level: 0 1

Perimeter: 81 cm 108 cm

Go a few more levels. Can you see a pattern? Can you find a formula? Do you think the perimeter will grow to infinity?

5) The Area of the Snowflake

In working on the area, remember that the areas of similar figures are proportional to the squares of their side ratios. For example, take two squares, one 5 cm on a side, the other 10 cm on a side. The area of the bigger one will be *four* times the area of the smaller. It will probably make your work easier if we say the area of the level one triangle is 1, and then describe the increasing area in terms of that. Here's how your function chart might start:

Level:	0	1
Area:	1	$1\frac{3}{9}$

Go a few more levels. Can you find a pattern? A formula? Does it seem like the area will grow to infinity?

6) Research

Can you learn how fractals were first discovered? Can you learn something about the people who discovered them? The most famous fractal is probably the Mandelbrot Set. See what you can learn about it. Find out how people are using fractal geometry in many different ways. Here are just some: computer game graphics, understanding stock markets, and understanding the distribution of galaxies in the universe.

7) Have fractal fun!