Back to our Roots

The Princeton Friends School opened in 1987 and on 9/14/1987 Richard Fischer handed out the first Problem of the Week. This problem, which examines square grids, is one that also occurred to John when he was sitting in the Meeting House, staring out of a window. Let's take a look at the roots of POW and add to it a bit as we examine the year's first, and the school's first, problem of the week.

1) How many squares are there in the pattern to the left? (Don't forget the 2 X 2 and 3 X 3 Squares.)
2) How many squares (of all sizes) are there in an 8 X 8 checkerboard?
3) How many squares (of all sizes) in a 20 X 20 checkerboard? (Is there a shortcut?)

That's where the original POW ends, but I think we can go a little bit further given the fact that they only had a single week to work on this problem, and we may have up to three. Let's take the original POW and expand upon that last phrase (Is there a shortcut?) a bit.

4) Can you discover a formula to accurately predict how many squares there will be in a checkerboard of any given size?

5) How about rectangles? How many rectangles (of all sizes) are there in the 3 X 3 grid?

6) How many rectangles (of all sizes) are there in an 8 X 8 checkerboard?

7) Can you discover a formula to accurately predict how many rectangles there will be in a checkerboard of any given size? What makes this problem more complicated than problem number 4?

8) Speaking of checkerboards, if we follow the checkerboard pattern of coloring in every other square so that no two blank squares touch and no two colored squares touch, what will be the ratio of blank to colored squares? Does this ratio change based on the size of the board? Compare a 3X3 and 4X4 square. What do you notice about the layout of the board as it grows?

9) How many squares are there (of all sizes) on a 1000 X 1000 board?

10) Have Fun!