Summer Recreation!

I. The Famous Locker Problem

A school has a hallway with 100 lockers. One morning, student #1 walks in and opens every locker. Then student #2 walks in and closes every second locker (locker #2, #4, etc.). Then student #3 comes in and *changes the state* of every third locker. That means if it's open she closes it, and if it's closed she opens it. (So, for example, she closes locker #3, which she finds open, but she opens locker #6, which she finds closed.) Then student #4 changes the state of every fourth locker. And so on, until student #100 changes the state of locker #100. Here's the question: At the end of this story, which lockers are open?! There are many ways to solve this. Making a smaller problem at first, with perhaps just 20 lockers, may help. Acting it out or keeping a record of changes on graph paper may help too. Good luck! Explain your answer.

II. Cheryl's Birthday

This problem was part of the Singapore and Asian Schools Math Olympiad. It became popular on the internet this spring. Albert and Bernard want to know when Cheryl's birthday is, and she gives them a list of possible dates, as follows:

May		15	16			19
June				17	18	
July	14		16			
August	14	15		17		

Cheryl tells Albert just the month of her birthday and tells Bernard the day (but not the month). Then, after they think for a while, this conversation happens:

Albert: I don't know when Cheryl's birthday is, but I'm sure Bernard doesn't know either.
Bernard: At first I didn't know when Cheryl's birthday is, but now I do.
Albert: Then I also know when her birthday is.

So... when is Cheryl's birthday? Explain all your steps!

III. Regular Tilings

How many patterns can you find that tile forever using two or more regular polygons? (The pattern that uses octagons and squares is probably the most familiar.) Can you present your solutions artistically?

IV. A Dinner Party

"My wife and I attended a dinner party with four other married couples. At the party some people shook hands. No one shook hands with himself (or herself), no one shook hands with his (or her) spouse, and no one shook hands with the same person more than once. When the party was over I asked everyone, including my wife, how many people they had shaken hands with. To my surprise everyone gave me a different number. How many people did my wife shake hands with?"

This problem was devised by Lars Bertil Owe of Sweden. It's a great problem because it seems impossible, yet if you keep using what you know, you can solve it! Drawing a diagram can be a big help! How can you show or explain your solution?

VI. Squares of Primes

Pick any prime number greater than three. Square it. Now divide that number by 12. What remainder did you get? Try it with other primes. What happens? Explain!

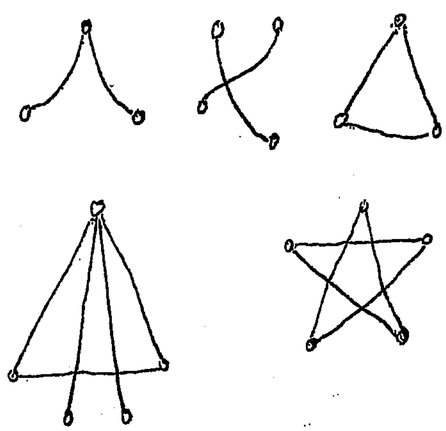
VII. Thrackles!

Thrackles were invented by John Conway, professor of mathematics at Princeton University. A thrackle is a doodle in the plane made up of a finite number of *paths* and *spots*. Each path has two distinct endpoints called spots. No path may cross itself. Every pair of paths must connect exactly once, either at a common endpoint or at an interior point where they cross. So no path can pass through a spot, and no paths can touch without crossing.

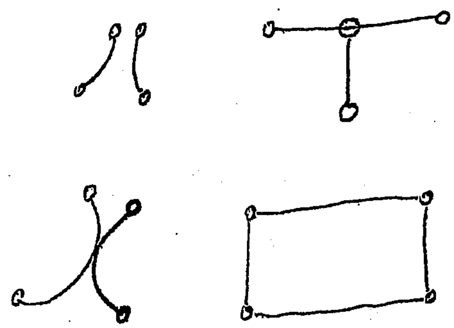
The question: Can there be more paths than spots? This is an open question. No one has solved it. No one has created a thrackle with more paths than spots, and no one has found a proof that there can't be one. John Conway has offered $1000 for the first correct solution!

Study the examples below. Draw some thrackles. What can you discover? Of course it's not too likely we'll solve a problem that has stumped John Conway and many other mathematicians for decades. But at our level, just drawing a valid thrackle is an accomplishment! As your thrackles get more complicated, you should make a table to check that every path is connected to every other path (just once!) in the thrackle. I put an example of my own at the bottom. (If two paths connect at a spot, I put an o; if they cross, I put an x.) Have fun!

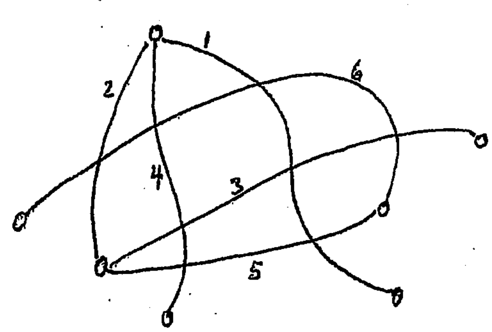
Valid Thrackles



Not okay!



Richard's Thrackle



	1	2	3	4	5	6
1		o	x	o	x	x
2	o		o	o	o	x
3	x	o		x	o	x
4	o	o	x		x	x
5	x	o	o	x		o
6	x	x	x	x	o	

Source for The Dinner Party: *Knotted Doughnuts* by Martin Gardner
Source for Cheryl's Birthday: http://en.wikipedia.org/wiki/Cheryl's_Birthday