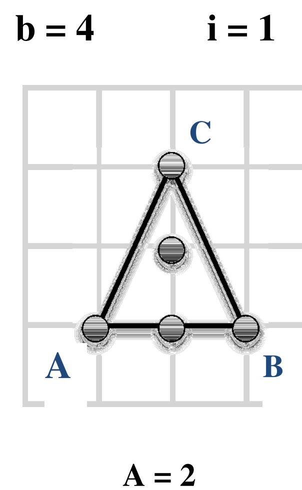
PFS 28-9 1/28/15 ______________________

Street Corner Polygons!

A polygon is a shape like a triangle, a rectangle, a pentagon, etc. For example, a hexagon is a polygon with six sides and six vertices (or corners). If we use graph paper, we can draw special polygons making sure that every vertex is at a "street corner" or lattice point of the grid. And we can have fun counting the street corners that are part of the polygon.

For example, look at the drawing on the right. We've made a triangle with a base of two units of length, and a height (or altitude) of two units. Each vertex of the triangle is at a lattice point of our graph paper. We've counted the number of street corners on the border of our polygon and found 4. (Three are the vertices of the triangle, and one is the midpoint of side AB.) So we write b = 4, using b for border. We’ve also counted the number of lattice points inside our polygon and found 1. So we write i = 1, using i for interior. Finally, we found the area of our triangle two ways, by using the formula A = (b x h)/2, and by comparing the triangle to a 2 x 2 square. And we've written A = 2.



Now it's your turn! Draw the street corner polygons described below on graph paper. For each one, indicate the number of border latice points, the number of inside (or interior) lattice points, and the area of the polygon. In addition, explain how you found the area!

1) A 2 x 2 square.

2) A triangle with a base of 3 and a height of 2.

3) A 4 x 3 rectangle.

4) A parallelogram that's not a rectangle with a base of 4 and a height of 3.

5) An isosceles trapezoid with bases of 4 and 2, and a height of 2.

6) A hexagon with two interior points.

7) A polygon that you invent!

The next two need to be drawn on a coordinate plane:

8) Connect the following points in cyclic order (H to E, E to X, etc.) to form a hexagon: H (-2, 0), E (-1, 3), X (3, 3), G (4, 0), O (4, -2), N (2, -2).

9) Draw pentagon ABCDE, with B = (6,1), C = (6, -2), D = (-3,-2), E = (-3,1), and the slope of EA = 1, and the slope of AB = -1/2.

10) Now look over all your work. Can you find a formula connecting A, b and i? Is there a way to get the area of each polygon just by counting the points on the border and the points inside?

Hint: the area will depend on two variables, b and i, the street corners on the sides of the polygon and the street corners inside it. To discover a formula you might want to control one variable and vary the other. For example, study this family: 1 x 1 square, 1 x 2 rectangle, 1 x 3 rectangle, etc. In that family there are no interior points and just the border points are increasing. Or study this family of isosceles triangles, all with a base of 2, and heights of 2, 3, 4, etc. Here just the number of interior points is increasing. Brain storm! Collaborate! Good luck!

11) And of course, have fun hanging out on those street corners!