PFS 28-14 4/29/15 ____________________________

Adding Fractions--*A Lot* of Fractions!

Adding fractions is fun! So why not have an infinite amount of fun? What happens if we try to add an infinite number of fractions? For example, consider this series with the simple rule that each term equals one half:

Example Series A: $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots$

It's pretty clear that if we go on forever, this sum will be infinite or, as mathematicians say, this series *diverges*. We can, however, compute partial sums for this series. Here's the beginning of a function table showing the term number, the fraction, and the sum of the terms so far, as both fractions and decimals:

n	nth term	cumulative total as a fraction	cumulative total as a decimal
1	$\frac{1}{2}$	$\frac{1}{2}$	0.5
2	$\frac{1}{2}$	1	1.0
3	$\frac{1}{2}$	$\frac{3}{2}$	1.5

But not every series goes to infinity. Here's one that doesn't:

Example Series B: $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\ldots$

This time our rule is that each numerator equals three, and each denominator is ten to the first power, ten to the second power, and so on. Here's how its function chart would start:

n	nth term	cumulative total as a fraction	cumulative total as a decimal
1	$\frac{3}{10}$	$\frac{3}{10}$	0.3
2	$\frac{3}{100}$	$\frac{33}{100}$	0.33
3	$\frac{3}{1000}$	$\frac{333}{1000}$	0.333

Mathematicians say this series *converges,* that is, they say that the infinite series has a finite sum. So, do you know what the sum of this infinite series will be? (You should!)

Your Assignment

For students in 4th and 5th grade, please choose at least one of the series below. (Series #1 is a good first choice for 4th and 5th graders.) For older students, please choose at least two. Make a function chart like the ones shown above, but go to at least the fifth term. Add the fractions using paper and pencil, and show your work! You can change fractions to decimals using a calculator if you wish. After you do your numerical work, write a paragraph or two to describe your work and your ideas. Some of the things to include in your writing: the rule for your series, what you notice about your series, whether you think your series will converge or diverge, how you worked on it and who you collaborated with, things you're curious about, etc. If you think your series converges, what do you think its sum will be? Please make sure all your work, numerical and written, is very neat and will look beautiful on the bulletin board! Please proofread your work! And remember, if you include research in your writing, cite your sources.

Series #1: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$

Extra things to think about for Series #1: Can you relate this series to a paradox by Zeno of Elea? Can you express this series as a Base Two "decimal"?

Series #2: $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\ldots$

This series is called the harmonic series. Mathematicians who worked on it included Nicole Oresme and the brothers Jacob and Johann Bernoulli.

Series #3: $1+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\ldots$

Do you recognize the denominators? A mathematician who worked on this one was Gottfried Leibniz.

Series #4: $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\ldots$

You do recognize the denominators of this one, don't you? This one is called the Basel Problem (named after a city in Switzerland) and was worked on by Leonhard Euler.

Finally, just to inspire you and pique your curiosity, here's an example using adding and subtracting:

Example Series C: $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}+\ldots$

The mathematician Leibniz was able to prove that this series converges, and that its sum is equal to pi divided by four! (That's approximately 0.7853981634.)

Have an infinite amount of fun!