PFS 28-4 10/23/14 _____

Square Numbers

"…gli spazii passati esser tra di loro come i quadrati e i tempi…" -- Galileo

On the right is the beginning of a function chart showing the first four square numbers. You can find square numbers using color cubes or graph paper, or by multiplying a number by itself.

n	n^2
1	1
2	4
3	9
4	16

1) On a separate paper make a neat function chart of the first ten or twenty square numbers.

2) Memorize the first ten or twenty square numbers!

3) How many square numbers are there? Explain.

4) How many numbers must be added each time, to go to the next square number? For example, what do we need to add to go from 1 to 4, from 4 to 9, etc.? Can you explain this?

5) Use graph paper to show how each square number is the sum of two triangular numbers. (For example, 16 = 10 + 6.)

6) Look at the ones column digits of your square numbers. What can you discover?

7) Is 3,496,587,477 a square number? Why or why not?

8) What is the one thousandth square number?

9) If you multiply two square numbers together, what kind of number will you get? Explain.

10) Count the number of factors each square number has. Now count the number of factors some non-square numbers have. What do you notice?

11) Can you add two square numbers to get a third square number? Three numbers that fit this formula ($a^2 + b^2 = c^2$) are called a Pythagorean triple. There is a formula for finding "all" of them. Learn how to use it and find some Pythagorean triples your math teachers might not know!

12) Learn the Pythagorean Theorem from geometry. Note that if the sides of a right triangle are all whole numbers, they will form a Pythagorean triple. Make a neat drawing of a right triangle with sides 3 cm, 4 cm and 5 cm. Draw the square on each side and label the areas. Write an equation connecting the areas.

13) Can you find a square number that is double another square number? (This might be impossible! Can you find a proof that it is?)

14) Study the numbers that are one less than a square (3, 8, 15 etc.). What can you discover about them?

15) What happens when you square a binomial, such as $(x + y)$?

16) Here is a nice theorem, first stated by Bachet, and proved by Fermat and Lagrange: Every positive integer can be expressed as the sum of not more than four squares. For example:

$1 = 1$ $\quad\quad$ $7 = 4 + 1 + 1 + 1$
$10 = 9 + 1$ $\quad\quad$ $14 = 9 + 4 + 1$

Explore this!

17) Which numbers are the sum of two square numbers? (For this project it's useful to consider zero as a square number.) Here are some numbers that are the sum of two squares:

$1 = 0 + 1$ $\quad\quad$ $2 = 1 + 1$
$4 = 0 + 4$ $\quad\quad$ $5 = 1 + 4$
$8 = 4 + 4$ $\quad\quad$ $13 = 4 + 9$

Can you find some patterns or rules? What if you focus on which *primes* can be formed as the sum of two squares?

18) Test a result that goes back to the Indian mathematician Brahmagupta (598-668). Multiply together two numbers that are the sums of two squares. Is the product also a sum of two squares?

19) 50 is the sum of two squares in two different ways! ($50 = 25 + 25$ and $50 = 49 + 1$.) Can you find more numbers like that? I think there are nine of them between one and 200.

20) A certain teacher liked to see his students do projects in large groups. If 3 students finished a project together, he'd give each child 3 candies. If 5 finished a project together they'd get 5 candies each. One day the boys did a project together and the girls did a project together. So the teacher had to give out $b^2 + g^2$ candies. The next day the boys and girls finished a project together. How much more candy did the teacher give out that day? (From *Algebra* by Gelfand and Shen.)

21) Galileo was one of the founders of modern science. While trying to understand gravity mathematically (and having fun rolling marbles down ramps) he made a discovery relating to square numbers. Report on this!

22) Graph the function $y = x^2$.

23) Have fun! $\quad$ 24) Learn a lot! $\quad$ 25) Work with someone!