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Inverses!

There are many ways to think about inverses. One way is to think about it as a way to get home. Here is Bob describing how he drives to his friend Clara's house:

"I make a right out of my driveway, drive exactly four miles east on Main Street, then make a left into Clara's driveway."

1) Write out directions for Bob for going home from Clara's house.

2) Write down your own set of directions. Then write the inverse directions. Can you find a general rule for writing down reverse trips?

We can think of operations being inverses of each other. For example, subtraction is the inverse of addition.

3) What is the inverse of subtraction?

4) What is the inverse of multiplication?

We can also think of numbers being inverses of each other. For example, -7 is the additive inverse of 7. (We usually call additive inverses *opposites.*) Opposites are pairs of numbers that add to zero.

5) What is the opposite of -34?

6) What number is its own opposite?

The multiplicative inverse of 2 is ½. (We usually call multiplicative inverses *reciprocals*.) Reciprocals are pairs of numbers that multiply to one.

7) What is the reciprocal of ¾?

8) What is the only number that has no reciprocal? Why doesn't it have one?

9) How can you use reciprocals to prove that the number of points between zero and one is equal to the number of points between one and infinity?

Another way to think of inverses is with functions. A one-to-one function has an inverse, but a many-to-one function will not.

10) What is the inverse of $y = 2x$?

11) Does $y = x^2$ have an inverse? Explain. Does the answer depend on the domain of the function?

12) What is the inverse of $y = 3(x - 2) + 7$?

The usual instruction for finding the inverse of an algebraic equation is "switch x and y and solve for y." Can you also use order of operations to do the same job? Think of it like driving directions.

13) Graph a linear function and its inverse. (For example, you could use $y = 3x$ and $y = x/3$.) Do the same job with another pair of inverse functions. Do you notice something about the graphs of a function and its inverse?

Inverses can come up in many places, and they don't have to be about numbers. What is the inverse of putting on your sneakers?

14) In English country dancing you might hear these directions for four dancers: "First corners change places; second corners change places; circle to the left half way." Can you find an inverse in those directions? Try it out!

When people began looking for new types of secret codes for computers and the internet, they began searching for "one-way functions." These are one-to-one functions that have inverses, but the inverse can be hard to compute. Here's an example, where the function is finding the product of two two-digit prime numbers. So the inverse would be factoring a number into its two prime factors.

15) What is the product of 19 and 83? Show your work. How long did it take you to get your answer?

16) What are the prime factors of 2479? Show your work. How long did it take you?

Now imagine that the prime numbers are a hundred digits long or longer. A computer can still multiply them very quickly. But finding the factors takes a long time, even with a computer, and this can help make a special kind of secret code.

17) Learn more about public key cryptography.

18) Have fun!