PFS 28-5 11/6/14 _____

The Invention of Chess

There is an old story that goes as follows. A very wealthy Indian king loved the new game of chess, and he asked his servants to find the inventor of the game. When the inventor appeared at the palace, the king told him he had invented something wonderful—a game that would give people pleasure and improve their minds. "Ask for anything you want," said the king, "I am almost infinitely wealthy." The inventor said, "Your Majesty, I don't need any reward. But since you offer… Could you pay me one grain of rice for the first square on my chessboard, two grains for the second, four for the third, eight for the fourth, and so on, doubling the amount of rice each time, till every square is accounted for." The king urged him to choose gold or diamonds, but the inventor stuck with his request. "Very well," said the king to his servants, "bring a sack of rice from the kitchen."

1) How many squares are on a chessboard?

2) Begin a table that will show the number of grains of rice for each square and the cumulative total up to that square. Go to at least the 9^{th} square. Your table should begin as follows:

Square	Grains	Total so Far
1	1	1
2	2	3
3	4	7
4	8	15

3) Can you find a way to estimate the amount of rice the inventor is asking for? Explain your estimate and how you got it. If your estimate is just a number of grains, help us picture how much rice that is!

4) Can you find the exact number of grains of rice the inventor is asking for? Can you help us picture how much rice that is?

5) The numbers in the second column of our table are called powers of two (1, 2, 4, 8, 16…). Do you notice something about the numbers in the third column? Do you see a short-cut for adding up the first n powers of two? (For example, what's an easy way to add 1 + 2 + 4 + 8 + 16?) Compare the total grains of rice the inventor would get for the first eight squares with the amount he would get just for the ninth square.

6) This story is an example of exponential growth. Can you find other examples of exponential growth and explain them? There are many examples in biology, finance, physics, etc.

7) The number of grains for each square could be represented as an algebraic function, $y = 2^n$. Can you graph this exponential function? Do you think you should use a different scale on your y axis from the one on your x axis? Why?

8) Judy's neighbors were away for the month of June, and Judy noticed there was some algae on the surface of their swimming pool on June 1st. Each day the algae covered twice as much area. On June 30th it finally covered the whole pool! On what day in June did it cover half the pool?

9) Some of you will remember that a perfect number is a number that equals the sum of its factors smaller than itself. Six is the first perfect number because 6 = 1 + 2 + 3. The second perfect number is 28, because 28 = 1 + 2 + 4 + 7 + 14. Now look at our table for the invention of chess. Look at row 2. What is 2 x 3? Look at row 3. What is 4 x 7? Can this help you find the third and fourth perfect numbers? Check that your answer is right by finding all the factors and adding them! (Hint: 8 x 15 will *not* give you a perfect number. Can you explain why?)

10) The powers-of-two short cut for finding perfect numbers goes back to Euclid, who wrote *The Elements* in about 300 BCE. And people are still using this short cut today to find very large perfect numbers by linking computers around the world in a project called GIMPS. Do some research and learn more about Euclid or GIMPS.

11) There are several versions of a game called nim. Here's one version. Put some objects (counters, pennies, blocks) in three piles. Players take turns taking any number of objects from any one pile. (You must take at least one and you can take the whole pile.) The person who takes the last object wins. There is a strategy to help you win that uses powers of two. Can you learn it?

$2^3 + 2^2$) Have fun!