PFS 28-7 12/12/14 _____

A Winter Wonderland of Puzzles

1) John Conway's Family Puzzle

John says, "I have sons and daughters. Each of my daughters has an equal number of brothers and sisters. Each of my sons has twice as many sisters as brothers. How many sons and daughters do I have?"

Can you solve John's problem? A good strategy might be trial and error. Pick different numbers and see if they work. If you're working as a class or with a group of friends, maybe you can act it out. Explain your solution!

2) Counting those Threes!

How many numbers between one and a hundred have a three in them? Okay, how many numbers between one and a thousand have a three in them? ("Have a three" means have at least one three.)

3) Fault-Free Rectangles

Arrange dominoes into a rectangle so no straight-line crack runs from one side to another.

4) Bracelets

Jody wants to make hoop bracelets with five beads on each. (Each bead will be a different color, and Jody has beads that are red, blue, green, yellow and white.) How many different bracelets can be made? "Different" means that if your bracelet is in the lost-and-found, you can recognize it as yours and no one else's. Can you draw (or make) the whole set?

5) Scrambled Labels

You have three boxes. One contains two pennies, one contains two dimes, and one contains a penny and a dime. Each box has been labeled, but someone switched the labels, so all the boxes are now labeled wrong. You may draw out one coin at a time from any box. What is the minimum number of coins you need to draw out and look at in order to re-label all the boxes correctly?

This problem is usually about white and black marbles, but we'll use coins at school. You don't get any information about the other coin when you reach in to take the first coin from a box.

6) The Dark and Stormy Bridge

It was a dark and stormy night. A family had to get to safety across a rickety old bridge. They have only seventeen minutes before the flood destroys the bridge. And they have only one flashlight, which they must use to cross the bridge. Grandpa takes ten minutes to cross the bridge; Mom takes five minutes; Brother takes two minutes; Sister takes one minute. No more than two people can cross at a time, sharing the flashlight. Can they all make it to the other side in seventeen minutes? Explain how. Is there more than one solution? If you think it's impossible, explain why. (No gimmicks, such as throwing the flashlight back! Someone has to bring the flashlight back to the people who are still waiting to cross.)

7) Romeo and Juliet

Romeo and Juliet live in neighboring castles. Unfortunately, they've been grounded! Fortunately, there is a messenger. Unfortunately, the messenger will steal whatever they send, unless it's in a locked box. Fortunately, Romeo has a box. The box has a big ring on it, and so does its lid, so it can be locked with a padlock. Fortunately, Romeo has a padlock and key, and so does Juliet. Unfortunately, Romeo's key opens only his own lock, and Juliet's opens only hers. How can Romeo send Juliet that special Valentine's present?

8) An Ancient Problem

Abu Ali al-Hasan ibn al-Haytham (sometimes called Alhazen) was born in 965, probably in Basra, Persia (now Iraq). He died in 1040, probably in Cairo, Egypt. He wrote more than 90 works on math and science; 55 of them have survived. Here is a math problem from al-Haytham:

Find a number that gives remainder 1 when divided by 2, 3, 4, 5 or 6; but when divided by 7, there is no remainder.

Al Haytham probably made up this problem to illustrate a law he discovered: If p is a prime number, then (p-1)! + 1 is divisible by p. This law was given the name "Wilson's Theorem" in 1770, and proved by the Italian/French mathematician Lagrange in 1771. Find other examples of that theorem!

9) A Simple Multiplication

Here's a straightforward problem! For some reason, not too many students have solved it!

116,415,321,826,934,814,453,125 x 8,589,934,592

10. How Many Children

"I hear some youngsters playing in the backyard," said Jones, a college math student. "Are they all yours?"

"Heavens, no," exclaimed Professor Abbott, the great number theorist. "My children are playing with the children from three other families, although our family is the largest. The Browns have a smaller number of children, the Carters a still smaller number, and the Drakes have the smallest number."

"How many children are there all together?"

"Well," said Abbott, "there are fewer than 18 children, and the product of the numbers of children in the four families happens to equal my house number."

Jones took out her notebook and pencil and started working. After a while she said, "I need more information. Is there more than one child in the Drake family?"

As soon as Abbott replied, Jones smiled and gave the correct number of children in each family. Knowing the house number, and whether the Drakes had more than one child, and being a good college mathematician, Jones was able to solve this puzzle. Believe it or not, YOU can solve this problem, and give the correct number of children in each family, just with the information given (and a little hard work). Good luck! (This problem was devised by Lester R. Ford.)

11) Happy New Year!

Is 2015 a prime number? If not, what are its factors? What's the next year that will be a prime number? Our school started in 1987. Was that a prime number? How many of the years of our school's history have been prime numbers? Explain how you got your answers!

12) Have fun!