PFS 27-2 10/1/13

The Seven Bridges of Konigsberg

"In the town of Konigsburg there is an island called Kneiphof, with two branches of the river Pregel flowing around it. There are seven bridges crossing the two branches. The question is whether a person can plan a walk in such a way that he will cross each of these bridges once but not more than once."

-- Leonhard Euler, 1735

The townspeople wanted to find a way to stroll over each bridge once, but so far they couldn't find a way to do it. They appealed to the great mathematician Euler for help. He solved the problem, and in so doing he started the branch of math called graph theory. In working on the problem Euler represented each land mass as a point or *vertex*, and each bridge as a line segment or *edge*.

1) If there is a solution to the challenge, find it and explain it!

2) If it can't be done, explain why.

3) Can you state a general rule or method?

4) Create a map or drawing with five to eight bridges where you can walk over each one once.

5) Create a map or drawing with five to eight bridges where you can cross each bridge once and get back to your starting place. A cycle!

A related (but more difficult) problem in graph theory is to find a path that visits each vertex exactly once. That is called a Hamiltonian path, in honor of the mathematican William Rowan Hamilton, who worked on this problem. If you get back to your starting place, it's called a Hamiltonian cycle.

6) Create a drawing or graph that has a Hamiltonian path or cycle. See if your friends can find the path or cycle! Can you create one where a Hamiltonian path is impossible?

7) Have fun!