Random Walks!

Random walks are fun things to study in math, and they have many applications. To quote from Wikipedia, "For example, the path traced by a molecule as it travels in a liquid or a gas, the search path of a foraging animal, the price of a fluctuating stock and the financial status of a gambler can all be modeled as random walks..." (http://en.wikipedia.org/wiki/Random_walk)

1) Roll a die 100 times, and keep a record on a 10 by 10 grid. For odd numbers record "R" and for even numbers "L". If you work in a team of four people, each person can roll a die 25 times, and you can string your results together.

2) How many R's did you get? How many L's? What was your longest string of R or L?

3) Now put a little person on a number line, start at zero, and move one unit right or left for each R or L. You can use a little paper clip on a number line, or, if you make a big enough number line, use yourself! Where do you end up after the 100th move? How far from zero are you?

4) For those who love Pascal's Triangle! Let's say you made two moves. All the possibilities are RR, RL, LR, and LL. So how many ways could you end up at 2? How many ways could you end up at 0? How many ways could you end up at -2? Notice something? This can tell us the probabilities of ending up at different points. (By the way, what's the probability of ending at 1 in two moves? Why?) Can you do the same thing for three moves? Four moves? More?

5) We've been working with a one-dimensional random walk, but we can make a two-dimensional graph of it by using "time" as our x-axis. Make an x-axis from 0 to 100, and a y-axis based on your results in #3. Graph your results. What's the farthest from zero you ever got? How many times did you cross zero?

6) Compare your answers to #3 with other students in your class. Find the average of everyone's final distance from zero. How does that number compare to 100--the number of moves we had?

7) For the ambitious! Try a two-dimensional random walk! This time your person will move on a coordinate plane, starting at (0,0). When you roll the die, use 1 for Right, 2 for Up, 3 for Left, 4 for Down, and if you get a 5 or 6, roll again. Keep a record of your moves. Then, as you move your person on a coordinate plane, keep a record of the coordinates for each move. (For example, 1, 4, 1, 3, 2, 2 on the dice would translate to R, D, R, L, U, U. So our guy would start at (0,0) and move to (1,0), (1,-1), (2,-1), (1, -1), (1,0), and (1,1).)

8) Try to learn something about how random walks are used by scientists and economists!

9) Have fun!