Number Necklaces!

        This is a great problem! It's easy to get into, but hard to understand completely. Here's how it goes:

        You have a whole lot of beads numbered 0 to 9. You may choose any two numbers to start your necklace (including choosing two of the same number). Then you add them to see which number to put on your necklace next. If the sum is a number bigger than 9, you use its ones digit to get your next bead.

        Here's an example. Begin with 2-2. So the next will be 4 (2+2), then 6 (2+4). But then 4+6=10, a two-digit number. So we use the 0 from 10 to get our next number, 0. The beginning of this necklace will look like this: 2-2-4-6-0-6-6-2. (Do you see where that last 2 came from?)

1) Choose two numbers and make a necklace. When you get the same two numbers again, in the same order, snip them off. Your necklace is now complete. How many beads are in your necklace?

2) Would a necklace starting with 4-6 be different from the one starting with 2-2? Explain.

3) Must a necklace cycle around to the same two numbers in the same order? Why or why not? Could a necklace go on forever without coming back to its starting numbers?

4) How many starting pairs of numbers can be made from the numbers 0 to 9?

5) Can you make other necklaces different from your first one?

6) Can you find a complete solution to this problem, that is, find all the possible necklaces? What are the lengths of your necklaces? What do you notice about them?

7) Present all your work neatly, so it will look attractive and readable on the bulletin board. Be sure your name is neat and prominent!

8) Can you make up a similar problem?

9) Have fun!