PFS 25-7 1/10/12 _____

Fun with Factors!

1) Chart

Make a chart by copying these rows and going to at least 20. Write a few sentences about overall patterns in the chart.

n	factors of n	# of factors	comments
1	1	1	unit
2	1, 2	2	first prime
3	1, 3	2	prime
4	1, 2, 4	3	square number

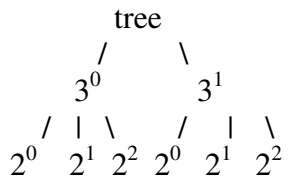
2) Divisibility rules!

Well, as the air traffic controller said to the pilot, "How's divisibility up there?" Anyway, there are short cuts that can help you decide if one number is a factor of another. For example, a counting number has five as a factor if, and only if, it ends in five or zero. How many rules can you find out and write down? Give an example or two for each.

3) Counting and Listing all the Factors

How can we find out how many factors a big number has? How can we list them all? Try this method: First make a factor tree of the number and write its prime factorization using exponents. Then add one to each exponent and multiply those numbers together. That will give you the number of factors. Then make a tree of the possible ways to combine those prime factors, and you'll have all the factors. See below for a simple example. (I skipped the factor tree.)

$12 = 2^2 \times 3^1$ $3 \times 2 = 6$ So 12 has six factors.



factors: 1 2 4 3 6 12

Try this method with one or more big numbers. Here are a few to try: 360; 400; 700; 6,125. Or choose one of your own.

4) The Famous Locker Problem

A school has 100 students and 100 lockers in a row. One morning student #1 enters the school and opens every locker. Then student #2 comes in and closes locker #2, #4, #6, etc. Then student #3 comes in and "changes the state" of locker #3, #6, #9, etc. Then student #4 comes in and changes the state of locker #4, #8, #12, etc. And so on, up to student #100, who changes the state of locker #100. Which lockers are open at the end of this whole story? Hints: A good strategy for solving this problem is "solve a simpler problem." Try using graph paper to work out the story for just ten or twenty lockers. Another good strategy is to think locally, rather than globally. Imagine you're a locker! Which students will open or close your door? Good luck!

5) Perfect Numbers

Perfect numbers are those whose factors (other than the number itself) add up to the number. For example, six is the first perfect number because $1 + 2 + 3 = 6$. It's easy to look up perfect numbers and get info on them, but first try to find the next perfect numbers by yourself. Hint: The second perfect number is smaller than 30. After you find the second perfect number, RAF will give you hints to help you find the third. Can you find the fourth on your own?? Learn more about perfect numbers and their history. Learn about GIMPS! Maybe become part of it!

6) Euclid's Algorithm

The great Euclid devised a way to find the greatest common factor of two numbers by doing a series of long divisions. First divide the smaller into the larger. Take the remainder and divide that into the divisor. Repeat, until you get a remainder of zero. When you do, your divisor is the GCF of the two numbers. Try it out! Learn more about Euclid!

7) Co-prime and Euler's Phi Function

Mathematicians have a few ways to say the same thing. For example, five and nine are co-prime, are relatively prime, have no common factor, have GCF = 1. Euler's phi function counts the number of numbers smaller than a given number that are co-prime to it. For example, six has two smaller numbers co-prime to it, one and five. Euler gave a formula for calculating his function. Learn it and give some examples. Can you explain why it works?

8) An "Impossible" Problem

What is the probability that two counting numbers, picked at random from the infinite set of counting numbers, will be co-prime? This may seem like an impossible problem, but there is a way to answer it, and the answer is quite surprising! See what you can learn about it!

9) Visible Points

Mr. Zero,Zero said, "It's easy for me to know if a and b are co-prime. It just means I can see them from my home (0, 0) without any lattice point blocking my view." What is Mr. Zero,Zero talking about? Can you draw some diagrams to illustrate this? Hint: Draw a line segment from (0, 0) to (8, 12). Then do the same from (0, 0) to (8, 15).

10) Have fun!