PFS 25-13 5/2/12 ______________________

Frieze Patterns

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 3 1 2 2 1 3 1 2 2 1 3 1 2
2 2 1 3 1 2 2 1 3 1 2 2 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 3 2 2 1 4 2 1 3 2 2 1 4 2 1
1 2 5 3 1 3 7 1 2 5 3 1 3 7 1
1 3 7 1 2 5 3 1 3 7 1 2 5 3 1
1 4 2 1 3 2 2 1 4 2 1 3 2 2 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

1) Study the two frieze patterns above. Write down anything you notice. It might help to think of the ones as borders and study the numbers enclosed by ones. Can you discover a rule about any four numbers that form a diamond? For example:

3
2 5
3

Hint: multiply east times west and north times south, then compare the products.

2) Work in a group and create a frieze pattern. Use the diamond paper. Start with a row of ones. Then zig and zag down as you wish, going four to six rows. Make your bottom row a row of ones. Now use the diamond rule to fill in your frieze. By working in a group you can check each other's work and discuss things you notice.

3) Here's another—surprising!—way to make a frieze pattern. Draw a polygon, for example an octagon. Draw in diagonals, without letting any cross, until your polygon is divided into triangles. Now label each vertex with a number counting how many triangles that vertex is part of. These numbers, reading around the polygon, can be a second row for a frieze pattern. (For example, the simple frieze pattern at the top is based on cutting a pentagon into three triangles.) Try making a frieze pattern this way, and show your "triangulated" polygon too!

4) How is it that our frieze patterns never require us to use a fraction to make our diamond rule come true? (I don't know the answer to this myself!!)

5) Try making a frieze pattern starting with a row of zeros, and using the rule that east *plus* west is one greater than north *plus* south.

6) Can you turn your arithmetical frieze into a work of art by using a different color for each number, or in some other way?

7) What type of symmetry does your frieze pattern show? Is the symmetry of the adding patterns different from the symmetry of the multiplying ones?

8) What if we used multiplication to make a frieze pattern like this:

```
1   1   1   1   1
  x   x   x   x
1   1   1   1   1
```

What number would x have to be? How is this related to the sides and diagonals of a square?

9) What if we used multiplication to make a frieze pattern like this:

```
1   1   1   1   1
  y   y   y   y   y
y   y   y   y   y
  1   1   1   1   1
```

What number would y have to be? How is this related to the sides and diagonals of a regular pentagon?

10) Can you learn more about frieze patterns?

Have fun!

Big thanks to John Conway, who introduced me to frieze patterns a few weeks ago—but didn't give me many answers!