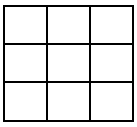
PFS 25-9 2/14/12 _____

Adding Squares

Guess what! This was our first Problem of the Week, back on September 14, 1987. (Back then I prepared the POWs with pen and paper, not on a computer.) I thought we should revisit this problem in honor of our 25th Anniversary, and in honor of Math Day, which is coming up on March 3. And of course, because it's a fun problem!



So here's the problem. How many squares, of all sizes, are in the drawing above? "Of all sizes" means you have to count the 1 x 1 squares, the 2 x 2 squares, and the one 3 x 3 square, and then add those numbers up to get your total.

The next thing I asked my students twenty-five years ago was to answer the same question for an 8 x 8 checkerboard, or for a 20 x 20 one. Is it possible to find a short cut?

Notice that we're counting square shapes, but also the numbers we're adding are square numbers. If you're looking for a pattern, a good idea is to make a function chart like the one on the right. S stands for "the sum of the first n square numbers."

n	S
1	1
2	5
3	14

If you're good at research and algebra, perhaps you can find an algebraic formula for this function, and demonstrate it in your write-up.

Do you remember Pascal's Triangle, where each number is the sum of the two numbers above it? Look on the right to see how it begins. If you look on the diagonals you should be able to find one with counting numbers, one with triangular numbers, and one that gives the sums of the first n *triangular* numbers.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
```

Here's a variation on Pascal's Triangle. It follows the same adding rule, but starts with 2,1. Can you use it to find the sums of the first n square numbers? (Check the diagonals.)

```
              2     1
            2   3   1
          2   5   4   1
        2   7   9   5   1
      2   9  16  14   6   1
```

Have fun!