PFS 24-8 1/20/11 _____

Regions in a Circle

Let's put points on a circle and connect each point to every other point with straight lines. The lines are like walls that make rooms or regions. How many regions will we get?

One point leaves one big region. Two points create two regions. Three points create four regions. What will happen with four points? With more?

1) Make some diagrams with circles, points on the circles, and straight lines connecting each point on the circle to every other point on the circle. Count the number of regions you get for each number of points on the circle. With six or more points, be sure to use large full-page diagrams. Use a protractor to draw neat circles and to measure arcs. Use a ruler for your straight lines. Work neatly please!

2) Make a chart like the one shown below. How far can you get? Check your chart with a friend.

points	regions
2	2
3	4
4	8

3) Can you find a rule or formula to help you understand this function?

4) Does it matter if the points are equally spaced around the circle?

5) Can you prove or explain any formula you find?

6) Can you find out how many regions 10 points will lead to? 20 points? 100???

7) Can you create a work of art using one of your diagrams?

Hints for the ambitious: This problem is related to the rule for the number of diagonals in a polygon with n sides. Can you find that formula? There is also a formula that relates all the points, segments and regions (vertices, edges and faces) in any network. Can you find that formula?

8) Have fun!

PFS 24-8 Alternative 1/20/11 _____

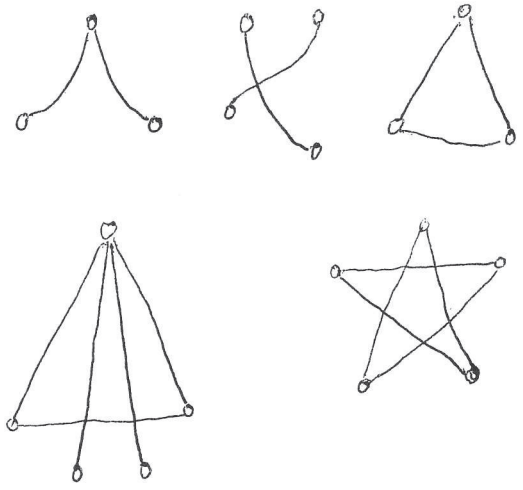
Yay! More Thrackles!

Thrackles were invented by John Conway, professor of mathematics at Princeton University. A thrackle is a doodle in the plane made up of a finite number of *paths* and *spots*. Each path has two distinct endpoints called spots. No path may cross itself. Every pair of paths must connect exactly once, either at a common endpoint or at an interior point where they cross. So no path can pass through a spot, and no paths can touch without crossing.

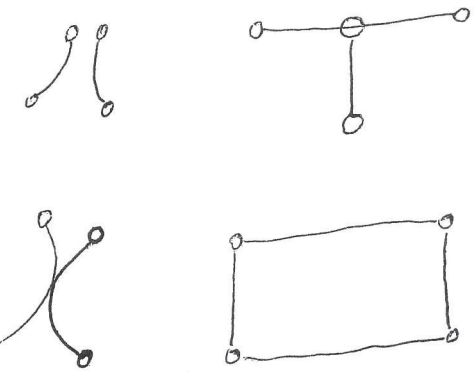
The question: Can there be more paths than spots? This is an open question. No one has solved it. No one has created a thrackle with more paths than spots, and no one has found a proof that there can't be one. John Conway has offered $1000 for the first correct solution!

Study the examples below. Draw some thrackles. What can you discover? Of course it's not too likely we'll solve a problem that has stumped John Conway and many other mathematicians for decades. But at our level, just drawing a valid thrackle is an accomplishment! As your thrackles get more complicated, you should make a table to check that every path is connected to every other path (just once!) in the thrackle. I put an example of my own at the bottom. (If two paths connect at a spot, I put an o; if they cross, I put an x.) Have fun!

Valid Thrackles



Not okay!



	1	2	3	4	5	6
1		o	x	o	x	x
2	o		o	o	o	x
3	x	o		x	o	x
4	o	o	x		x	x
5	x	o	o	x		o
6	x	x	x	x	o	

Richard's Thrackle