PFS 25-5 verso 11/21/11 _____

Goldilocks and the Square Root of Two

Goldilocks heard about "irrational numbers" in school, but she didn't believe it! Any number on the number line has to equal some fraction, she thought to herself. The teacher said the square root of two was irrational, but Goldilocks decided she could prove her teacher wrong, with just a little trial and error.

Well, she thought, it's between one and two, so I'll start with one and a half. And soon she had a chart that looked like this:

mixed number estimate	improper fraction estimate	estimate squared	comment
$1\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{4} = 2\frac{1}{4}$	Too big!
$1\frac{2}{5}$	$\frac{7}{5}$	$\frac{49}{25} = 1\frac{24}{25}$	Too small!
$1\frac{5}{12}$	$\frac{17}{12}$	$\frac{289}{144} = 2\frac{1}{144}$	Too big!

1) Continue Goldilocks' chart for three more rows.

2) Change her estimates and "estimate squared" numbers into decimals.

3.1415926535897923...) What do you notice about this chart?

4) Do you think Goldilocks will find a fraction equal to the square root of two if she keeps trying?

5) Look at the second column. How is she finding improper fractions that come so close to being the square root of two? Is she following a rule for the next improper fraction? What is it?

6) Do mathematicians have any *proof* that no fraction can ever equal the square root of two? Can you learn a proof?

7) Hmmm... I just tried $\frac{41}{29}$ on my calculator and got 1.413793103. That doesn't seem to repeat, and maybe it goes on forever. Could $\frac{41}{29}$ be an irrational number? Explain!

8) Why are irrational numbers called irrational?

9) Have fun!