

An Old-Fashioned Grocer

(I first learned this story and lesson from the mathematician Bill Thurston, who was then teaching at Princeton University. His daughter Emily was one of the founding students of PFS, and Bill helped us start our Problem of the Week program.)

There was an old-fashioned grocer who carried his goods in a big wagon pulled by horses. At the back of the wagon he had a big two-pan balance for weighing the things he sold. For example, he might sell someone 7 pounds of flour, or 23 pounds of potatoes. This grocer wanted to be able to weigh out any whole number of pounds from one to forty. Of course he could have carried a one pound weight, a two pound weight, a three pound weight, etc. But those forty weights would add up to a lot of weight to put in his wagon!

1) How much weight would that be? In other words, how much is $1 + 2 + 3 + 4 + \dots + 39 + 40$? Is there a short cut for that? Can you find a short cut by trying examples with fewer numbers to add? Is there a name for numbers (like 1, 3, and 6) that you get by adding the first so many counting numbers?

Well, it turns out this grocer was a good mathematician too! He needed only four weights, that added up to forty pounds total, to help him weigh any amount from one to forty.

2) What four weights did he use? How did he manage to weigh any whole number up to forty with just those weights? Can you use pictures to show part of the answer? What special numbers was he using?

3) What if another grocer wanted to be able to weigh every whole number up to 121 pounds? How could she do that with the fewest weights and the least amount of total weight?

Have fun!