PFS 24-4 10/26/10 _____

The Return of Number Necklaces!

This is a great problem! It's easy to get into, but hard to understand completely. Here's how it goes:

You have a whole lot of "beads" numbered 0 to 9. You may choose any two numbers to start your necklace (including choosing two of the same number). Then you add them to see which number to put on your necklace next. If the sum is a number bigger than 9, you use its ones digit to get your next bead.

Here's an example. Begin with 2-2. So the next will be 4 (2+2), then 6 (2+4). But then 4+6=10, a two-digit number. So we use the 0 from 10 to get our next number, 0. The beginning of this necklace will look like this: 2-2-4-6-0-6-6-2. (Do you see where that last 2 came from?)

1) Choose two numbers and make a necklace. When you get the same two numbers again, in the same order, snip them off. Your necklace is now complete. How many beads are in your necklace?

2) Must a necklace cycle around to the same two numbers in the same order? Why or why not?

3) How many starting pairs of numbers can be made from the numbers 0 to 9?

4) Can you make other necklaces different from your first one?

5) Can you find a complete solution to this problem, that is, find all the possible necklaces? What are the lengths of your necklaces? What do you notice about them?

6) Can you explain why this problem breaks up into a certain number of necklaces of certain lengths? [I'm still working on this myself! -- RAF] You might want to see how this problem shapes up in Base Two, Base Three, etc.

7) Present all your work neatly, so it will look attractive and readable on the bulletin board. Be sure your name is neat and prominent!

8) Can you make up a similar problem?

9) Have fun!

Ruth-Aaron Numbers

On April 8, 1974, Hank Aaron, of the Atlanta Braves, hit his 715th major league home run. This broke a baseball record that had lasted since 1935: 714 home runs by Babe Ruth. The numbers 714 and 715 had been on many people's minds. Carl Pomerance, then an assistant professor at the University of Georgia, noticed something when he looked at the prime factorizations of these numbers.

1) Write down the prime factorizations of both numbers. What do you notice about which prime numbers are involved?

Pomerance shared his discovery with a fellow teacher, David Penney. That teacher mentioned it in his class, and one of his students, named Jeremy Jordan, made *another* discovery.

2) Add the prime numbers in each factorization. Write down what you notice!

Pairs of consecutive numbers with this second property are called Ruth-Aaron numbers. We're going to find more of them. But notice that there can be two ways to count prime factors. Think of a number like 12. 12 = 2 x 2 x 3. If we're counting *distinct* prime factors, we'd say the prime factors of 12 are 2 and 3, and their sum is 5. If we're counting *repeated* prime factors, we'd say the prime factors of 12 are 2, 2, and 3, and their sum is 7.

3) Counting distinct prime factors, find some more Ruth-Aaron pairs. Hint: there are four such pairs smaller than 100. Write a sentence or two explaining how you worked on this.

4) Counting repeated prime factors, find more Ruth-Aaron pairs. Hint: there are four such pairs smaller than 100. Describe how you found them.

5) Can you explain why some pairs of numbers are on both lists?

Carl Pomerance published a paper about Ruth-Aaron numbers, and the great mathematician Paul Erdos got interested in them. This led to their working together on 21 papers. (Erdos also worked on highly composite numbers.)

6) Can you learn more about Babe Ruth, Hank Aaron, Carl Pomerance or Paul Erdos?

7) Can you find other pairs of consecutive numbers that together use the first n primes in their factorizations?

8) Have fun!