PFS 23-6 1/28/10 _____

The Pythagorean Theorem

1) In a right triangle the two shorter sides next to the right angle are called the legs. The longest side, opposite the right angle, is called the hypotenuse. Draw a right triangle with legs of 8 cm and 15 cm. Do the numbers 8 and 15 help you predict how long the hypotenuse is? Measure it! Now draw two more right triangles. Measure the legs. Can you predict how long the hypotenuse is in each triangle? Do the measurements.

2) The Pythagorean Theorem is a "secret formula" that connects the lengths of the sides in a right triangle. Find out what that secret formula is, and write it down in your own words! Give an example to explain it!

3) Write down the first twenty square numbers. See how often you can find two square numbers that add up to a square number. Write them down. Write down the square roots too. For example, you might write:

$36 + 64 = 100$ 6, 8, 10

If you draw a triangle with the square root numbers, what kind of triangle will you get?

4) You can understand the Pythagorean Theorem without numbers, as a statement about the areas of squares drawn on the sides of a right triangle. One way to see this was devised by an amateur mathematician named Henry Perigal. (It's both a puzzle and a proof of the Theorem.) Younger students may wish to cut out the pieces from the card-stock pattern given to you. Older students should create their own Perigal shapes. (Remember to have two copies before you cut one up!) Can you learn something about Perigal's life?

5) The Pythagorean Theorem can help us solve problems about right triangles. See if you can solve these:

a) The legs of a triangle are 7 cm and 24 cm. How long is the hypotenuse?

b) (This problem comes from an ancient Babylonian tablet.) A 30 foot ladder is upright against the wall of a house. If the top of the ladder slides down 6 feet, how far from the wall will the bottom of the ladder be?

c) How long is the space diagonal of a box that is 3 feet by 4 feet by 12 feet?

6) To use the Pythagorean Theorem we have to be able to find square roots. Nowadays a calculator can help us do that. But is there a way to find square roots with paper and pencil? Sure! We can use an iterative procedure where we estimate, divide, average, and repeat. For example, let's say you want to find the square root of 70. Well, you know it's a little bigger than 8, so use 8 as your first estimate. Do 70 divided by 8 and get 8.75. So the square root of 70 is between 8 and 8.75. Average 8 and 8.75, and get 8.375. That's your new estimate, and

now repeat, dividing 70 by 8.375. Use this method to find the square root of 17. How precise an estimate can you get? Have fun dividing! Show all your work neatly!

7) The three sides of a right triangle will not always be measured in whole numbers. (What are the smallest three whole numbers that can make a right triangle?) One or more sides might have a fractional part. For example, 1.5 cm, 2 cm, and 2.5 cm will make a right triangle. But sometimes you might have a side that's not a whole number, and that's not equal to any (proper or improper) fraction either! (How is that possible??!!) Learn about the isosceles right triangle and the 30-60-90 right triangle, and the *irrational numbers* that go with them. When did people first discover irrational numbers? How can we know a number is irrational? Can you find other triangles with irrational numbers?

8) Three whole numbers (like 6, 8, 10) that make a right triangle are called a Pythagorean Triple. If the numbers have no common factor (like 3, 4, 5), they're called a primitive Pythagorean Triple. There's an infinite number of these, and there's a secret formula that can help you find all of them. Pick two numbers, p and q, with p bigger than q, no common factor between p and q, and one odd and one even. Now write down (p^2-q^2), $2pq$, (p^2+q^2). You should have a primitive Pythagorean Triple! How many can you find? (If you use a little algebra, you should be able to prove that this secret formula will always give a Pythagorean triple. It's a little harder to prove that every Pythagorean triple will be in this form.)

9) You can use the Pythagorean Theorem on the coordinate plane. For example, how far apart are the points (-3, -5) and (6, 7)? Find the distance between some other points? Can you use the Pythagorean Theorem to write a distance formula between any two points?

10) You can use some algebra to prove the Pythagorean Theorem. Draw a square of, say, 7 inches. Divide each side into a shorter (left hand) part and a longer (right hand) part, for example, 2 and 5 inches, and label those segments a and b. Now connect the four points where segments a and b meet to get a slanty square inside your original square, and label the side of this slanty square c. You now have four identical right triangles with sides a, b, and c. Use your knowledge of area and your ability to square (a + b) to prove the Pythagorean Theorem.

11) Find out what a logical *converse* is. Is the converse of a true statement necessarily true? Is the converse of the Pythagorean Theorem true? Can you use it to find out whether sides of 65, 72, and 97 would make a right triangle?

12) Do you like ancient history? What can you find out about Euclid's Book I? About Plimpton 322? About YBC 7289?

Have fun!