PFS 23-7 3/1/10 _____

Points and Lines

I. Planting Trees

Mr. Cedar came home from work and told his daughter, "I had an interesting job today. I planted trees in five rows of four each."

His daughter Laurel, who's in third grade, said, "Oh, Daddy, I know how many trees you planted! You planted twenty trees."

Mr. Cedar said, "Laurel, I'm very proud of you. You're doing so well in math and really learning your times tables. But actually I planted only ten trees."

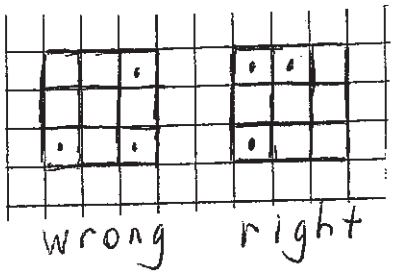
"But, Dad, how can that be? Five times four is twenty."

"Well, I'll give you a hint. Look at something your teacher put at the top of one of your wonderful math papers." Laurel looked and figured it out. Can you?

II. Four Lines

Arrange nine dots in a 3 x 3 square grid in the middle of a page. Can you draw four straight lines without lifting your pencil, so each dot will have a line through it?

III. Different Distances



Can you place three dots in the centers of three squares of a 3 x 3 checkerboard, so every pair of dots is a different distance apart? At right is shown one wrong way and one right way. Not counting reflections and rotations, there are four other solutions. Can you find them? In how many ways can you do this job for four dots on a 4 x 4 checkerboard? Try this problem for higher numbers. Warning: seven dots on a 7 x7 board is the highest for which a solution exists, and there is only one solution for the 7 x7 board. To think about how hard it might be to find, how many different ways are there to place seven dots on a 7 x 7 board? You might want to work on this with checkers or chess pieces on a chess board. Good luck!

IV. Thrackles

Thrackles were invented by John Conway, professor of mathematics at Princeton University. A thrackle is a doodle in the plane made up of a finite number of *paths* and *spots*. Each path has two distinct endpoints called spots. No path may cross itself. Every pair of paths must intersect exactly once, either at a common endpoint or at an interior point where they cross. So no path can pass through a spot, and no paths can touch without crossing.

The question: Can there be more paths than spots? This is an open question. No one has solved it. John Conway has offered $1000 for the first correct solution!

Study the examples on the Thrackle page. Draw some thrackles. What can you discover? Would this problem be different on a sphere or donut? Have fun!

PFS 23-7 Alternative 3/1/10 _____

The Likeness Sequence or Die Gleichniszahlen-Reihe

Many years ago Jane F. gave me (RAF) a math problem I couldn't solve. She showed me this:

1
11
21
1211
111221

And she asked me what the rule was, and what the next few rows would be. After a few years I gave up, and Jane explained it to me. As they say, "It's so easy, even a child can do it." Give yourself a few minutes to study the pattern. Even if you can't figure out the rule, can you write down some things you notice? If you do figure out the rule, write it down! If you don't figure it out on your own, ask Dave or Richard (or Jane) for a hint. And if the hint isn't enough, ask us for the rule! Then write out the next few rows. Check your work as you go!

1) Write the first ten (or more) rows.

2) What are some things you notice?

3) What do you notice about the last digit in each row? The last two digits in each row? Can you explain or make a proof?

4) Can a 4 ever appear in this pattern?

5) How do the rows seem to be growing in length? Can you make a table of the lengths of each row, and the ratios of each row to the preceding row? (For example, row five is 6 digits long and 3/2 (or 1.5) times as long as the previous row.) What do you notice? How long do you think the 20th row will be? The 30th row?

6) What happens if you start this whole sequence with a different seed? For example,

2	or	13
12		1113
1112		3113

What are some things you notice about *your* pattern?

7) There is one seed that will not allow the sequence to grow. Can you find it?!

Have fun!