Highly Composite Numbers

The great Indian mathematician Srinivasa Ramanujan lived from 1887 to 1920. He was unusual in that he was mostly self-taught in math, yet he was already making discoveries as a teenager. Ramanujan was the first to discuss and work on *highly composite numbers.* A highly composite number is a counting number that has more factors than any number smaller than itself.

1) Make a chart of the first several highly composite numbers. For each one include its prime factorization, a list of all its factors, and its total number of factors. Note that mathematicians include 1 and 2 as the first highly composite numbers, even though 1 is a unit and 2 is prime. Your chart should begin something like this:

HCN	prime factorization	all factors	total number of factors
1	1	1	1
2	2	1, 2	2
4	2^2	1, 2, 4	3
6	2 x 3	1, 2, 3, 6	4
12	2^2 x 3	1, 2, 3, 4, 6, 12	6

2) What do you notice on your chart?

3) Will all highly composite numbers after 1 be even? Why?

4) Why isn't there a highly composite number with 5 factors?

5) 4 is a power of two. Will any higher power of two be a HCN? Explain.

6) Guess what! If you have the prime factorization of a number, there's a short cut to finding the total number of factors without finding all the factors! Find the shortcut!

7) Are all highly composite numbers factorials? Are all factorials highly composite numbers?

8) 720,720 and 1,081,080 are HCNs. Where are those repeated digits coming from?

Have fun!

Square Numbers

On the right is the beginning of a function chart showing the first four square numbers. You can find square numbers using color cubes or graph paper, or by multiplying a number by itself.

1	1
2	4
3	9
4	16

1) On a separate paper make a neat function chart of the first ten or twenty square numbers.

2) Can you memorize the first ten or twenty square numbers?

3) How many square numbers are there? Explain.

4) How many numbers must be added each time, to go to the next square number? For example, what do we need to add to go from 1 to 4, from 4 to 9, etc.?

5) Look at the ones column digits of your square numbers. What can you discover?

6) Is 3,496,587,477 a square number? Why or why not?

7) If you multiply two square numbers together, what kind of number will you get? Explain.

8) Can you find a square number that is double another square number? Caution! This might be impossible!

14) Study the numbers that are one less than a square (3, 8, 15 etc.). What can you discover about them?

Have fun!