PFS 23-10 5/6/10 _____

Base Two and Nim!

Our number system is a *place value* system. 4,102 does not mean seven (4 + 1 + 2). 4,102 means (4 x 1000) + (1 x 100) + (0 x 10) + (2 x 1). Our system is based on the number ten. The value of each column is the next power of ten. We can have place value systems with other bases. The most important is *base two.* Modern computers do their "thinking" in base two. The number thirteen in base two would be written 1101_{two}. Can you explain why?

1) Write the powers of two from 1 to 1024. Try to memorize them!

2) You can count to 1023 on your ten fingers! Make paper rings labeled with powers of two from 1 to 512. Put 1 on your right pinky, 2 on your right fourth finger, and so on. Now practice counting in base two by touching certain fingers to the table. For example, if you touch your right pinky and right index finger, that would equal nine. Your left thumb plus your right index finger plus your right fourth finger would equal 42.

3) Write the numbers from 0 to 31 in base two. Keep neat columns.

4) What do you notice about base two numbers as you write them in order?

5) Change 87_{ten} to a base two number. Change 111011_{two} to a base ten number.

6) Explain in your own words how to convert from base ten to base two, and from base two to base ten.

7) Add the following numbers *in* base two: 10110 + 1111.

8) If children grew up in a base two world, what times table would they have to learn? Write it out in full. Comment!

9) Can you find out *why* computers do their thinking in base two? Also, try to learn about ASCII code, which is a standard way computers represent letters and numbers. For example, a capital C would be represented as 01000011_{two}, which equals 67_{ten}.

11) The first two perfect numbers are six and twenty-eight. Write out these numbers in base two. Does this help you find the next perfect number? Remember to check by adding the factors! (This is a shortcut, but it ain't *that* short!)

12) Learn something about the history of base two, which goes back to ancient India, China and Africa--among other places!

13) Learn to play Nim! Start with three piles of counters, any number in a pile. Two players take turns. On your turn you must remove at least one counter, and you can take as many as you like, but only from one pile. The player who takes the last counter loses. The secret to winning is to count each pile in base two. (For example, think of a pile of thirteen as 8 + 4 + 1.) Can you learn more about how to use base two strategy to help you win? Good luck! Have fun!