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Monochords!

One of the most important ideas in science is that there are mathematical laws hidden in nature which human beings can discover. Thousands of years ago people found mathematical patterns in two important areas. The first area was astronomy, where people learned about the seasons, the phases of the moon, the eclipse cycles, and even the precession of the equinoxes! And the second area was music, where people learned that familiar musical intervals were organized by simple mathematical ratios.

But wait! What's a "ratio"? It's such an important idea that there are many ways to think of it. A ratio can be thought of as a fraction, a rate, a proportion, a percentage, or a comparison between two numbers by multiplying or dividing.

1) Play with your monochord! Can you make a tune? Can you find it again? Do you notice anything about the notes you get and the fraction of your string that is vibrating?

2) Mark your monochord at the halfway point, and to show thirds and quarters. What musical intervals can you discover if half your string is vibrating? Or two thirds? Or three quarters? Can you explore further with fractions?

3) Notice the frets on a guitar. Are they evenly spaced? Can you mark some "frets" on your monochord so you can play a scale? What is a musical scale anyway?

4) Can you play music with a friend or in a group? Can you harmonize? Does musical harmony relate to mathematical ratios?

5) Again, from very early times, people connected astronomy and music in their thinking about patterns in nature. They imagined that the planets sang tunes as they moved. This is sometimes called "The Harmony of the Spheres." Here's how a character in a play by Shakespeare described it:

How sweet the moonlight sleeps upon this bank!
Here will we sit, and let the sounds of music
Creep in our ears; soft stillness and the night
Become the touches of sweet harmony.
Sit, Jessica; look, how the floor of heaven
Is thick inlaid with patines of bright gold:
There's not the smallest orb which thou behold'st
But in his motion like an angel sings,
Still quiring to the young-eyed cherubins:
Such harmony is in immortal souls;
But, whilst this muddy vesture of decay
Doth grossly close it in, we cannot hear it.

Merchant of Venice V. i.

Learn more about the Harmony of the Spheres. (The great astronomer Johannes Kepler, who wrote The Harmony of the Worlds in 1619, was the first person to understand that planets move in ellipses, not perfect circles. He related the long axis and short axis of the ellipse to musical intervals, and claimed that the earth sings "mi fa mi". Can you play mi fa mi on your monochord? Learn more about Kepler!)

6) Galileo is often considered the "Father of Science." He lived from 1564 to 1642. Here's what he wrote about finding laws in nature:

"Philosophy is written in this grand book - I mean the Universe - which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles and other geometrical figures, without which it is humanly impossible to understand a single word of it."

Guess what! Galileo's father (Vincenzo Galilei) was a musician (a lutenist) and a music theorist, who made discoveries about vibrating strings! Learn more about Galileo and his father!

7) Have fun!

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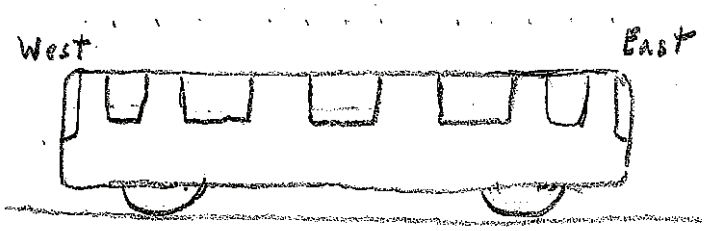
Some Classic Problems!

1) $3n + 1$

This is an unsolved problem! Start with any counting number. If it's odd, multiply by 3, then add 1, then divide by 2. If it's even, divide by 2. Now repeat. What happens as you start with different numbers? You get into a 4-2-1 loop, don't you? No one has found a starting number that doesn't lead to that loop, but no one has proved every number has to lead to that loop! Can you keep a chart of what happens with every starting number from 1 to 30? How many steps does each need till it enters the loop? Which starting number took the longest? Is there a pattern? Can you invent a similar problem?

2) The Bus

Which direction is this bus going? Hint: a four-year-old might be able to solve this problem!



3) Directions

You walk one mile south, one mile east, and then one mile north, and you're back where you started! How can that be? Where did you start? This classic problem has one classic answer. But guess what! There are many other possible answers! Can you find them?

4) Three Light Bulbs

You're in a big science building. In Room 101 there are three switches, marked A, B, and C, with "on" and "off" correctly labeled. In Room 109 there are three light bulbs on a lab table, labeled 1, 2, and 3. Each switch controls one bulb, but you don't know which controls which. You can't see into Room 109 from Room 101. How can you start in Room 101, do whatever you want with the switches, walk to Room 109 only once, and know which switch controls which bulb?

5) Regular Star Polygons

They have all sides and angles equal, but the sides are allowed to cross each other. A good way to draw them is to divide a circle into n equal parts, then "connect the dots." There are no star polygons for $n = 3$ or 4. Can you draw some beautiful star polygons? Can you make a set, for example, all the different star polygons for $n = 11$? How many different ones are there for each n? Is there a rule? What determines whether the star polygon is all connected, like the five-pointed star, or has parts, like the six-pointed star? Can you relate the star polygons with $n = 12$ to musical ideas, such as the circle of fifths?