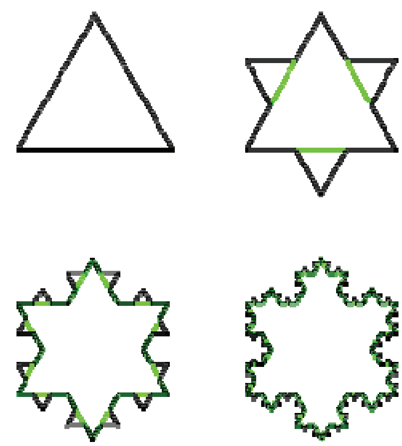
PFS 22-11 5/4/09 ________________

Fractals!

The Mandelbrot set is one example of a *fractal.* Here are some activities to help us learn more about fractal geometry.

I. The Koch Snowflake



The Swedish mathematician Helge von Koch first described the Koch snowflake in 1904. We start with an equilateral triangle, and at each step we add equilateral triangles facing out to the middle of every edge. Here are the first four steps.

The Koch snowflake is what we get if we iterate forever. See if you can draw the initial iterations. You might want to start with a triangle 9, 18 or 27 cm long. Can you make your snowflake beautiful? Can you keep a chart of the perimeter and area of your triangle at each step? If we continue forever, will the perimeter be infinite? Will the area be infinite?

II. The Sierpinski Gasket

The Polish mathematician Waclaw Sierpinski first discovered the Sierpinski gasket in 1915. Start with an equilateral triangle. (Edges of 16 or 32 cm might work well.) Mark the midpoint of each edge. Connect the midpoints and "remove" the middle triangle by coloring it. Now repeat with the three smaller equilateral triangles, and so on. Try using different colors at each level. How far can you go? What can you learn about the Sierpinski Gasket?

III. The Cantor Set

The German mathematician Georg Cantor first discovered the Cantor set in 1883. Here's how to make it. Start with all the points on the number line between 0 and 1 (and including 0 and 1). Now divide the segment into three equal parts, and remove the middle segment, leaving its endpoints 1/3 and 2/3. Now you have two segments. Iterate! Divide those segments into three equal parts and remove the middle segment (leaving its endpoints). Keep track of how much of the original segment you've removed. (So far, we've removed 1/3 + 2/9, or 5/9 all together.) If we continue forever, how much of the original segment will we remove? How many points will be left? Will 1/3 and 2/3 still be remaining? How about 1/9? How about 1/4? Can you learn more about the Cantor set?

IV. We Don't Need No Stinking Computers!

The amazing videos of the Mandelbrot set that we saw could not have been made without computers. But we can still learn about the Mandelbrot set using pencil and paper. You just need to know how to square a binomial, and remember that i^2 = -1. Just choose any complex number as your c. Always start with z = 0. Then calculate z^2 + c. That will be your new z. (Always keep c the same.) And iterate. Here's an example:

c = i
z= 0
0^2 + i = i
Our new z = i
i^2 + i = -1 + 1
Now z = -1 + i
(-1 + i) ^2 + i = -i
Now z = -i
(-i)^2 + i = -1 + i
We're back to z = -1 + i
So we're in a loop with two numbers, and we'll never escape to infinity; so the number i is part of the Mandelbrot set!

Here's a fact that can help you: once a point is more than two units from the origin, it will escape to infinity. Here are some points you might want to try as your c:
1
1 + i
-0.5 +0.5i
-2,
-0.3 + 0.2i

Choose your own points for c! Have fun!

V. Applications

Do some research and find out how people are using fractal geometry in many different ways. Here are just some: computer game graphics, understanding stock markets, understanding the distribution of galaxies in the universe, creating digital sundials.

Have fractal fun!