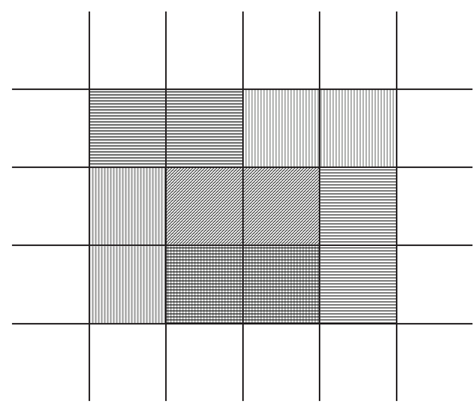
Street Corner Polygons!

A polygon is a shape like a triangle, a rectangle, a pentagon, etc. For example, a hexagon is a polygon with six sides and six vertices (or corners). If we use graph paper, we can draw special polygons making sure that every vertex is at a "street corner" or intersection point of the grid. And we can have fun counting the intersections that are part of the polygon.

For example, look at the drawing on the right. We've made a triangle with a base of two units of length, and a height (or altitude) of two units. Each vertex of the triangle is at an intersection point of our graph paper. We've counted the number of intersection points on the border of our polygon and found 4. So we write b = 4. And we've counted the number of intersection points inside our polygon and found 1. So we write i = 1. Finally, we found the area of our triangle two ways, by using the formula A = (b x h)/2, and by comparing the triangle to a 2 x 2 square. And we've written A = 2.



Now it's your turn! Draw the street corner polygons described below. For each one, indicate the number of border intersection points, the number of inside intersection points, and the area of the polygon. In addition, explain how you found the area!

1) A 2 x 2 square.

2) A triangle with a base of 3 and a height of 2.

3) A 4 x 3 rectangle.

4) A parallelogram that's not a rectangle with a base of 4 and a height of 3.

5) An isosceles trapezoid with bases of 4 and 2, and a height of 2.

6) A hexagon with two inside points.

7) A polygon that you invent!

8) On a coordinate plane, a pentagon ABCDE, with B = (6,1), C = (6, -2), D = (-3,-2), E = (-3,1), and the slope of EA = 1, and the slope of AB = -1/2.

9) Now look over all your work. Can you find a formula connecting A, b and i? Is there a way to get the area of each polygon just by counting the points on the border and the points inside?

Hint: the area will depend on two variables, b and i, the street corners on the sides of the polygon and the street corners inside it. To discover a formula you might want to control one variable and vary the other. For example, study this family: 1 x 1 square, 1 x 2 rectangle, 1 x 3 rectangle, etc. In that family there are no interior points and just the border points are increasing. Or study this family of isosceles triangles, all with a base of 2, and heights of 2, 3, 4, etc. Here just the number interior points is increasing. Good luck!

Have fun!

PFS 21-9 Alternative 2/12/08

____

Three Chess Board Problems

1) Place a knight on the corner of a 4 x 4 chess board. What's the greatest number of squares it can visit (including its starting point) by making knight moves and never visiting any square twice? (And of course make a record of your solution!)

2) Place a queen on a edge square next to a corner square of a standard chess board. Find a way the queen can pass through every square of the 3 x 3 square in that corner, making only four queen moves. (The queen can use the whole board and can visit squares more than once.)

3) Assume you have 32 dominoes and a chessboard with squares of matching size, so each domino can cover two squares of the board. The 32 dominoes can completely cover the chessboard. What if we remove one square from one corner, and one square from the diagonally opposite corner. Can 31 dominoes cover the board now? If so, show how! If not, explain why not!

Have fun!