PFS 21-7 1/7/08 _____

Probability Puzzles

Probability is expressed as a fraction. One way to get this fraction is to calculate it mathematically. The numerator is the number of "successes." The denominator is the total number of equally likely possibilities. For example, in rolling a die, the probability of getting a one is 1/6. The probability of getting an odd number is 3/6 or 1/2. Another way is to keep records. For example, what is the probability that Richard will tell a Brooklyn story during a particular math class? Keep track of all your math classes and make a fraction!

Please answer in complete sentences! Explain your answer and how you got it! Show your work!

1) What is the probability of getting a prime number when one die is rolled? What's the probability of getting a square number? A perfect number/

2) What is the probability of getting a seven when two dice are rolled? A twelve?

3) A penny and a nickel are tossed. What's the probability that they both land heads?

4) You flip a penny four times and get heads each time. What's the probability that the next toss will be heads?

5) What's the probability that it will rain during the school day on an January day in Princeton?

6) What's the probability that a PFS student will finish all of his or her lunch?

7) The Three Cards. You have three cards in a hat, one red on both sides, one white on both sides, one red on one side, white on the other. You draw one at random and look at only one side. It is red. What is the probability that the other side is red? Can you do this both mathematically and experimentally?

8) The Fifty Balls. There are 25 red balls and 25 white balls. You have to pick one blindfolded from one of two bowls. If you get a red one, you get a prize. You are allowed to decide how the balls are distributed among the two bowls, but you can't tell which bowl is which. How can you maximize your chances of winning?

9) RAF. The Royal Air Force was losing some planes to anti-aircraft fire, and they consulted a mathematician about adding reinforced metal to parts of their planes. The mathematician said to check all their returning planes and place a mark on a model plane wherever any plane had a bullet hole. He then looked at the model and said, “Reinforce your planes wherever there are *no* marks.” Was this good advice? Explain.

10) New Assigned Seats. The fourteen third graders are saying numbers at random to get new assigned seats. What’s the probability that no third grader will get his or her old seat again?

11) Mrs. Smith says, “I have two children, and my younger one’s a boy.” What’s the probability her other child is a boy? Mrs. Jones says, “I have two children, and one of them’s a boy.” What’s the probability her other child is a boy?

12) Two numbers are chosen at random from the infinite set of natural numbers. What’s the probability that they are relatively prime? (Reminder: "relatively prime" means they have no common factor other than 1.) This is a seemingly impossible problem that *can* be answered. Here's a strategy that can help: solve a simpler problem. For example, what's the probability that two numbers between one and ten are relatively prime? How about between one and one hundred? Another way to make it a simpler problem is this: what's the probability that any two natural numbers are not both divisible by two? How about not both divisible by three? If you make progress on this problem, you should consult someone who has studied calculus and knows a beautiful result by the great Swiss mathematician Leonhard Euler about the sum of the reciprocals of all the square numbers.

13) You choose an M&M at random from a bowl of M&Ms. What’s the probability it will be red? (Do this experiment at home!)

Have fun!

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Plain Old Calculations

Okay, enough of all this thinking! How about a little old-fashioned paper and pencil calculating practice? That should be fun and relaxing! Work neatly on your own paper. Write down any comments you wish to add!

1) Let's start with seven easy multiplication problems:

$142{,}857 \times 1$ $142{,}857 \times 2$

$142{,}857 \times 3$ $142{,}857 \times 4$

$142{,}857 \times 5$ $142{,}857 \times 6$

$142{,}857 \times 7$

2) How about using exponents and multiplying?

$2^5 \times 9^2$

3) How about using exponents and simplifying fractions? Here's an easy one and a harder one:

$$\frac{2^2 + 4^2}{1^2 + 3^2}$$

$$\frac{3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2}{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2}$$

4) Some long division practice? Change 1/29 to a decimal. (Remember to keep going till it stops or repeats!)

5) Subbing in! Sub in some whole numbers (for example, 1 to 41) for x in this expression. (Notice anything?)

$x^2 - x + 41$

6) And finally, another multiplication:

$8{,}589{,}934{,}592 \times 116{,}415{,}321{,}826{,}934{,}814{,}453{,}125$