PFS 22-3 10/8/08 _____

Fun for Young and Old!

Here are some problems that came—mostly—from our lower grades. Use this POW as an opportunity to set out and present your work very neatly, in a logical order, and to explain your thinking carefully and fully.

1) Wow! Some years ago the first-graders recently worked on this problem—and solved it! Give it a try! How many different ways can you make up a set of ten markers, if you have as many as you want of three colors, and if you use at least one of each color in each set? Examples: 1 red, 1 green, 8 blue, 3 red, 4 green, 3 blue.

2) The second-graders have been working on an old POW—about the number of letters in the name of a number. Pick a number. Write it in English. Count the letters. Now write that number. Continue. For example, start with *eighteen*. 8 letters. Now try the number *eight.* Five letters. Etc. What eventually happens? Explain! Try it in other languages!

3) Here's a follow-up question to #2. Does this kind of pattern have to lead to a fixed point or a loop? Could it go on forever without repeating? Explain.

4) The third-graders worked on this problem: How many different combinations of coins can equal 47 cents. (Example: 4 dimes, 1 nickel, 2 pennies.) See if you can do it!

5) Follow up to #4. What if you made a function chart that showed the number of cents and the number of ways to make it? It would start like this:

Number of cents	Number of ways
1	1
2	1
3	1
4	1
5	2

Question: In row one the number of cents equals the number of ways. Is this possible again in another row? When will it first happen again? (We found many ways to make 47 cents, but not 47 of them.) If you think it's not possible, explain why.

6) Goldilocks and the Square Root of Two

Goldilocks heard about "irrational numbers" in school, but she didn't believe it! Any number on the number line has to equal some fraction, she thought to herself. The teacher said the square root of two was irrational, but Goldilocks decided she could prove her teacher wrong, with just a little trial and error.

Well, she thought, it's between one and two, so I'll start with one and a half. And soon she had a chart that looked like this:

Mixed Number Estimate	Improper Fraction	Estimate Squared	Comment
$1\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{4} = 2\frac{1}{4}$	Too big!
$1\frac{2}{5}$	$\frac{7}{5}$	$\frac{49}{25} = 1\frac{24}{25}$	Too small!
$1\frac{5}{12}$	$\frac{17}{12}$	$\frac{289}{144} = 2\frac{1}{144}$	Too big!

1) Continue Goldilocks' chart for three more rows.

2) Change her estimates and "estimate squared" numbers into decimals.

3) What do you notice about this chart?

4) Do you think Goldilocks will find a fraction equal to the square root of two if she keeps trying?

5) Look at the second column. How is she finding improper fractions that come so close to being the square root of two? Is she following a rule for the next improper fraction? What is it?

6) Do mathematicians have any *proof* that no fraction can ever equal the square root of two? Can you learn a proof?

7) Hmmm... I just tried $\frac{41}{29}$ on my calculator and got 1.413793103. That doesn't seem to repeat, and maybe it goes on forever. Could $\frac{41}{29}$ be an irrational number? Explain!

8) Why are irrational numbers called irrational?

9) Have fun!