PFS 22-1 9/8/08 _____

Fibonacci Part I

Leonardo of Pisa, also called Fibonacci, was born in Pisa, in Italy, about the year 1170. His father was a diplomat in North Africa, and Leonardo traveled a great deal as a young man. In 1202 he was back in Pisa and had published his first book, *Liber Abaci.* It was the first book to introduce Hindu-Arabic numerals, the kind we use today, to Europe. In his book, Leonardo described his early life as follows:

When my father, who had been appointed by his country as public notary in the customs at Bugia [in North Africa] acting for the Pisan merchants going there, was in charge, he summoned me to him while I was still a child, and having an eye to usefulness and future convenience, desired me to stay there and receive instruction in the school of accounting. There, when I had been introduced to the art of the Indians' nine symbols through remarkable teaching, knowledge of the art very soon pleased me above all else and I came to understand it, and I learned whatever was studied by the art in Egypt, Syria, Greece, Sicily and Provence, in all its various forms.

1) Before *Liber Abaci,* people in Europe used Roman numerals to write numbers. Find out how Roman numerals work and write some numbers in Roman numerals.

2) Is the Roman numeral system a place value system? What does that mean? Why did people find the Hindu-Arabic system better? (By the way, does it really have "nine" symbols?)

3) The most famous math problem in *Liber Abaci* concerns some rabbits who reproduce according to mathematical rules. Here it is: Assume a "pair" always means a male/female pair, and that each pair, after they're two months old, will give birth to one new pair every month. (And assume none of the rabbits die.) If you start with a new-born pair on January 1st, how many pairs will you have at the end of a year? To help solve this problem, make a function chart like this:

month	number of pairs
January	1
February	1
March	2
April	3

4) The number sequence (1, 1, 2, 3, 5, 8...) that we get in solving the problem about rabbits has been named the Fibonacci sequence. (Fibonacci probably learned about this sequence in his travels; for example, it was studied by ancient Indian mathematicians.) It has many amazing properties. Can you state a simple math rule for continuing this sequence that has nothing to do with rabbits? What do you notice about the sequence? Is there a pattern for odd and even in the sequence? If you square a Fibonacci number, how will that compare to the product of its two neighbors? (For example, multiply 5 by itself, then compare that to 3 x 8.) Is there a short-cut for adding up the first n Fibonacci numbers?

5) The French mathematician Edouard Lucas who lived in the 19th century studied the Fibonacci sequence. He also invented a new sequence, called the Lucas sequence in his honor. It follows the same rule as the Fibonacci sequence, but it begins with 1, 3, 4, 7... Write out the beginning of the Lucas sequence under the Fibonacci sequence:

1, 1, 2, 3, 5...
1, 3, 4, 7, 11...

Compare and contrast them! Do you see any connections between the sequences when they're written next to each other?

6) The Fibonacci numbers can be found in nature, and are used in computer programming etc. Can you find out more about them?

7) Extra for those who like a big challenge: Later in his life Fibonacci received a challenge problem from another mathematician, Johannes of Palermo. (This problem was not made up by Johannes, but was taken from a book by Omar Khayyam, the Arabic mathematician and poet.) He challenged Fibonacci to find a solution to the equation:

$$x^3 + 2x^2 + 10x = 20$$

Fibonacci proved the solution was not an integer or a fraction and gave an approximation correct to nine decimal places. (But he gave his solution in base 60.) Can you do it?

Have fun!