PFS 21-2 9/25/07 _____

Sticks!

Sticks is a fun game you can play with a friend using just your hands. You may know the game with different rules. If so, please describe your version in your POW paper.

Sticks uses a kind of math called modular arithmetic, or "clock arithmetic." On an everyday clock we use only 12 numbers. If you start school at eight o'clock and stay for seven hours, we say you leave at three o'clock, not "fifteen o'clock." We could say $8 + 7 = 3 \bmod 12$. In Sticks we use mod 5. You might want to practice some mod 5 arithmetic before playing. For example, $3 + 1 = 4 \bmod 5$. And $4 + 3 = 2 \bmod 5$. You can picture the five numbers on a circle. Or you can think of the remainders you would get if you divided each ordinary number by five.

Here's how to play the game. Start with the index fingers on each hand pointing out. That represents "one" on each hand. Players take turns. On each turn you decide which of your opponent's hands to touch with one of your hands. Then your count is added to that hand. To keep count on each hand, you use a fist for "zero" and up to four fingers. You can't pass, or touch your opponent using a zero hand if you have one. If you have a zero hand, and an even number in your other hand, you can bump your hands together and split your total number between your two hands. (For example, you could change zero on one hand and four on the other to two on each.) Bumping doesn't count as a move. If you can get your opponent's two hands to zero, you win!

1) Play Sticks! Teach it to a family member. Can you discover some strategies? Do you know another version? Can you invent a variation?

2) Can you learn something about Carl Friedrich Gauss, the inventor of modular arithmetic?

3) Try making a times table mod 5. Make a five by five square and label the left side and the top from zero to four. Then fill in the products mod 5. For example, $3 \times 4 = 2 \bmod 5$. What do you notice or discover in your times table? Try one mod 7, or mod 13.

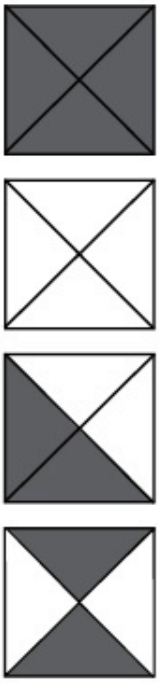
4) How are "odd and even" an example of modular arithmetic?

5) Have fun!

PFS 21-2 Alternative

A Different Kind of Color Square

Many of you know our color square puzzle. It is made of squares with two diagonals, forming four triangles on each square. Then the squares are colored using up to three colors.



Here's another kind you may enjoy exploring. It also uses squares with two diagonals, forming four triangles on each. This time, however, you should use *five* colors. On each square you may use one or two colors, and if you use two colors, there must be two triangles of each. Let's say your five colors included gray and white. Then you could have an all gray square, and all white one, and two different squares using a combination of gray and white. See the illustration on the right.

1) How many color squares will you have in your complete set? Explain how you got your answer!

2) Make a complete set! Be sure to put your name on the back of each square.

3) Try to arrange your squares into a bigger square so all the colors match up edge to edge. (Don't worry about the border!) If you find a solution, make a picture of it.

4) What else can you create with your new color squares?

5) Have fun!