PFS 20-8 1/29/07 ________________

Ramsey Theory

In 1930 the British mathematician Frank P. Ramsey published a paper that has led to a branch of math called Ramsey Theory. Here's one way to get started on understanding it:

1) Draw a circle with five equally spaced points on it. In pencil, lightly draw a segment connecting every point to every other point. You now have a regular pentagon, with all its sides and diagonals. In graph theory sides and diagonals are together called *edges.* Now choose two colors (for example, red and blue) and color each edge with one color.

2) Do you have three points in your graph that are connected with one color or the other, forming a triangle of one color? If so, make another graph with five points, and this time find a way to have *no* triangles with edges of all one color!

3) Now draw a circle with six equally spaced points. Do the same job as before, coloring every edge (how many are there now?) one of two colors. Do you have a triangle of one color or the other? Can you find a way to color the graph with six points so there won't be a triangle of one color or the other? Don't search too hard! It's impossible to find. Part of what Ramsey proved is that with six points we'll always have at least one triangle of one color or the other.

4) *Parties!* Here's another way to understand this part of Ramsey's Theorem. Let's say you invite six guests to a party. *You* know everyone. But some of your guests may not have met each other before. Let's call them *strangers.* Others may have met each other before. Let's call them *friends.* Then no matter which six people you invite to your party, there will be--at a minimum--either three people who are all friends to each other or three who are all strangers to each other. For a party with five guests, that need not be true. Throw some parties and test this out!

5) *Ramsey's Theorem as a Game.* Start with the six points on the circle. Player one draws a red edge. Then player two draws a blue edge. Play continues in this fashion. The first player to complete a triangle of his or her color loses. Can there be a draw? Play some games. What strategy do you come up with? How will the game be different if the first person to complete a triangle *wins*?

6) *Proving this part of Ramsey's Theorem*. How can we prove that our coloring of the graph with six points will always lead to at least one triangle of all one color? We could draw every possible coloring, and look at each one. Mathematicians call that a "brute force" proof. (In case you want to try it, there are 78 possible different colorings!) There is a simple and elegant proof, which also helps us understand *why* it's true. See if you can find it! Can you explain it in your own words?

7) *Getting bigger!* Now draw a (large!) circle with eighteen equally spaced points, and connect every edge. (Do you know how many edges that's going to be?!) Color each edge either red or blue (for example). This time we're looking for a quadrilateral of all one color, including its two diagonals. (In other words, four points that have all six connections all one color.) According to Ramsey's Theorem there must be one! But with seventeen points, you should be able to find a coloring with no "complete graph" of four points all one color. Good luck!

Ramsey numbers! A Ramsey number is the smallest integer that forces a complete graph to occur. For example, the Ramsey number for three red and three blue is six. The Ramsey number for four red and four blue is eighteen. Surprisingly, despite all the brilliant mathematicians on our planet, and all the high speed computers, no higher Ramsey number is known. Paul Erdos, a Hungarian mathematician who helped develop Ramsey Theory said the following about R(m,n), the Ramsey number:

> "Imagine an alien force, vastly more powerful than us, landing on Earth and demanding the value of R(5, 5) or they will destroy our planet. In that case, we should marshal all our computers and all our mathematicians and attempt to find the value. But suppose, instead, that they asked for R(6, 6). In that case we should attempt to destroy the aliens!"
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> Source: http://en.wikipedia.org/wiki/Ramsey's_theorem

8) Learn more about the life of Frank Ramsey or Paul Erdos. Report on either one. Here's a little quote about Ramsey by his brother:

> "Though we were at different schools, in holiday times we saw a great deal of each other and we spent a lot of time together hitting a tennis ball against the wall, the rudiments of squash rackets, or bowling a ball to each other in a wicket or that sort of thing: playing together, just us two, and talking a great deal about all sorts of things. He was interested in almost everything. He was immensely widely read in English literature; he was enjoying classics though he was on the verge of plunging into being a mathematical specialist; he was very interested in politics, and well-informed; he had got a political concern and a sort of left-wing caring-for-the-underdog kind of outlook about politics. I was aware that he was far cleverer than I was and knew much more, yet there was such a total lack of uppishness about him that we just conversed in a friendly way and he never made me feel inferior though I was so vastly below par intellectually, and that was the wonderful joy of it."
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> Source: http://en.wikipedia.org/wiki/Frank_P._Ramsey