Perfect Numbers

or

Who's Afraid of a Little Hard Work?!

In the Good Old Days (also known as the Bad Old Days) mathematicians got to play with numbers with paper and pencil[1] and do long, long calculations. As a result they got to see and understand number patterns we might not notice if we used a calculator. They were also very motivated to discover short cuts! Let's experience some of that with this POW!

A **perfect number** is a number equal to the sum of all its "proper factors" (factors smaller than itself). For example, 12 is *not* a perfect number because $1 + 2 + 3 + 4 + 6$ does not equal 12.

1) Find the first perfect number! Hint: it's smaller than 12. Make a chart of all the numbers from 1 to 11. For each one, list all its proper factors, and then the sum of its proper factors. Which one is perfect?

2) Find the seond perfect number. Hint: it's between 20 and 35. Use the chart method again.

3) A first short cut! Do you remember the triangular numbers, 1, 3, 6, 10, 15....? (We ran into them with our Ramsey Theory work.) Well, every (even) perfect number is a triangular number. Here's another hint: the third perfect number is between 400 and 550. This time save yourself time and make a chart with just the triangular numbers in that range. Which one is the third perfect number?

4) Another short cut! Every (even) perfect number has this form: it's a power of two multiplied by the sum of all the powers of two up to that one. We can see this if we make a chart with three columns: powers of two, sum of all the powers of two so far, product of columns one and two. Here's how the chart would begin:

1	1	1
2	3	6
4	7	28

---

[1] or papyrus and brush, or clay tablet and stylus!

So now, using this short cut, find the fourth perfect number!

5) Feeling intrepid? Want to find the fifth perfect number? Hint #1: it's smaller than fifty million! Hint #2: there's another short cut! Want to figure out *that* short cut? Think about why 120 (8 x 15) is *not* a perfect number. Good luck!

6) Be the first on your planet to find the first odd perfect number! (Or to prove that there are no odd perfect numbers!) Please post your answer in the Great Room no later than March 1, 3007.

7) Learn more about perfect numbers, their history, and the people who worked on them. If you want to be all modern, learn about GIMPS. Perhaps you and your family would like to participate in GIMPS.

8) Have fun!