5/30/06

Review and Reprise and Recessional (Revised)

Here are some challenges and unsolved problems from last year and this year.

1) Be the first (or second) on your block (or square)!

Can you discover your own 4 x 4 Graeco-Latin Square? Be the first on your block! The jacks, queens, kings and aces of a set of playing cards are an ideal way to look for one. Reminder: A Latin square is a group of n symbols, arranged in an n x n square, so one of each symbol is in each row and column. A Graeco-Latin Square consists of two Latin squares superimposed so one of each of the  $n^2$  ordered pairs appears just once. For example, here's a 3 x 3 Graeco-Latin Square:

 11
 23
 32

 22
 31
 13

 33
 12
 21

2) Fun in 3-D!

No one at PFS has found a way a 3-D tic-tac-toe game can end in a tie. Is it possible? Can you find a tie ending? Can you prove it's impossible?

## 3) Beat your calculator!

Can you find a better approximation for the square root of 71 than your calculator can give you? Reminder: start with an estimate, e.g. 8. Divide 71 by 8. Average the quotient and the divisor. Now you have a better approximation. Iterate!

4) All about bees!

A male bee has one parent--a mother (the queen). A female bee has two parents (male and female). Make a chart showing the number of ancestors a male bee has in each generation. Do the same for a female bee. What do you notice?

5) All about integers!

In how many ways can you write a number adding up strings of only ones and twos? For example, 1 = 1, so only one way. 2 = 1 + 1 and 2 = 2, so two ways. Jumping ahead, 4 = 1 + 1 + 1 + 1 and 4 = 1 + 1 + 2 and 4 = 1 + 2 + 1 and 4 = 2 + 1 + 1 and 4 = 2 + 2, so five ways. Make a function chart for this function. Can you find out how many ways we can write the number 10 using sequences of ones and twos?

## 6) A welcome change!

Can you go from WORK to PLAY (via POOL and FOOD if possible!) changing only one letter at a time, making sure every word in your chain is in the dictionary? GOOD LUCK! And while we're at it, can you go from GOOD to LUCK?

7) Cross those dots!

Can you arrange nine dots in a 3 x 3 square, and then cross them all out by drawing four straight lines, without lifting your pencil from the paper? Is there more than one solution?

8) Add those fractions!

What does 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 ... add up to? Will the sum go up to infinity as you keep adding fractions? Or is there a number the sum will never get beyond? Explain!

9) The Fraction Game!

Keane's amazing discovery has been the school record (and maybe the world's record!) for months! Here it is: 89/45 + 2/95 + 1/856 = 1.99999863... Can you beat it?

10) Patterns in Products of Primes

Multiply the first n primes. Then add one. What kind of number do you get, prime or composite? For example,  $2 \ge 3 + 1 = 7$ , a prime.  $2 \ge 3 \ge 5 + 1 + 31$ , a prime. So far only one student has found out on her own the first time this pattern leads to a composite. Can you find it by your own efforts?

11) Magic Square!

Can you make up your own original magic square?

12) A secret message!

ACB AAA BCA BCA CCA CAA CAC BBA BBA ABB BCC CCC

13) Have fun!