PFS 19-12 Alternative 4/24/06 _____

Prime Numbers

Our guest teacher, Allison, showed us Euclid's proof that the set of prime numbers is infinite. Let's review that and have some fun with it.

Prime numbers are numbers that have exactly two factors. Numbers greater than one that are not prime are called composite. One is called the unit.

Here's another way to think of it. If you have a prime number of square tiles, the only rectangle they can form is one unit wide, for example, 1 by 13. If you have a composite number, you can make other rectangles. For example, twelve will make 1 by 12, 2 by 6 and 3 by 4.

There are four prime numbers smaller than ten. (What are they?) But as you go up, primes become rarer and rarer. At first people couldn't be sure if there were an infinite number of primes, or if there was a highest prime number, and then no more. It seemed like an impossible question to decide, because how could you prove it? You can't write down an infinite list of primes. And if you think there's a highest prime, you can't check to infinity that there are no more!

The problem-solving strategy of *starting small* works well here. What if 2 and 3 were the only primes? 2 x 3 = 6, so 6 is divisible by both 2 and 3. But what kind of number can 7 be? It's one away from 6, so it can't be divisible by 2 or 3. Try the same process with 2, 3 and 5. This is a way to begin understanding Euclid's proof.

1) Make a list of all the prime numbers less than 100.

2) Explain how to check if a number is prime.

3) 2 x 3 +1 = 7, a prime number. 2 x 3 x 5 +1 = 31, another prime number. When is the first time this process will give us a composite number? What do you notice about its prime factors?

4) Write out Euclid's proof in your own words.

5) Students asked if there are patterns in the primes. Explore that on your own! One interesting way to work on it goes as follows: Arrange all the counting numbers in a "square spiral" on graph paper, then highlight the primes. What do you notice?

			7	9	10	11		
			6	1	2	12		
			5	4	3	13		
		18	17	16	15	14		

6) What else can you learn about prime numbers? Allison mentioned that primes are used in computer codes; can you learn more about that? What can you find out about twin primes, such as 11 and 13, that are two apart? Can you find out what the highest prime number known is?

7) Have fun!