PFS 19-7 1/5/06 _____

Polycubes!

Polycubes are the three dimensional version of Polyominoes. They are formed by putting cubes together face to face. When we use one, two or three cubes, the problem is essentially the same as with Polyominoes. (Why is that?) With tetracubes (made of four cubes) we encounter our first truly three dimensional shapes.

1) How many tetracubes are there? As with pentominoes, pieces that can be turned or flipped to look like each other are not counted as different. But mirror image shapes that can't be turned into each other are counted as different. Did you search for them in an orderly way? How? How can you be sure you've found them all?

2) Draw each tetracube.

3) There are 1390 ways to pack the tetracubes into a 4 x 4 x 2 box. How many can you find? How can you record your solution?

4) Find all the pentacubes! How many are there? Is there an orderly way to find all of them?

5) Draw all the pentacubes!

6) What more can you learn about polycubes?

7) Have fun!

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Three Stories and a Poem

I. The Inheritance

A Bedouin father died and left a will instructing his three sons to divide his wealth as follows: 1/2 to the oldest, 1/3 to the second, and 1/9 to the youngest. Their father owned seventeen Arabian horses, and the sons realized they couldn't carry out his wishes. (Why not?) Then the Sultan stopped by to offer his condolences. They told him of their problem dividing the horses. The Sultan said, "Your father was a wise man and a great friend of mine. I hereby give my horse to his estate!" The sons objected, but the Sultan insisted. Can they divide the horses now that there are eighteen? How many will each son get? They thanked the Sultan, who rode off on his horse! Explain how this was possible!

II. Stealing Apples

Out of six children it's known that exactly two have been stealing apples. But which two? When the children were questioned Ann said it was Betty and Charles. Diane said it was Edgar and Fred. Edgar said it was Fred and Betty. Charles said it was Ann and Betty. Betty said it was Edgar and Diane. (Fred couldn't be found when the questions were asked.) Four of the children who were questioned named one of the "criminals" correctly, and one incorrectly. The fifth one (not necessarily Betty!) named both incorrectly. Okay, who stole the apples?! (Hint: A diagram might help!)

III. An Old Trick Question

A salesman checks out of a hotel and pays $30 for his room. After he leaves, the desk clerk realizes he charged $5 too much, so he sends the bellhop after the salesman with $5. The bellhop is dishonest. He thinks, "If I give him $3, he'll be glad to get it, and I can keep $2." So that's what he does. So let's see, 30 - 3 = 27, so the salesman paid $27 altogether. The bellhop kept $2. $27 + $2 = $29. But the story started with a $30 payment. Where did the 30^{th} dollar go?!

IV. Trees in Rows

Your aid I want, nine trees to plant
In rows just half a score;
And let there be in each row three.
Solve this: I ask no more.

In 1821 John Jackson published this poem in a book called *Rational Amusement for Winter Evenings.* Another way to state this problem is this: arrange nine points on a paper so that there will be ten straight rows of three each. (It *is* possible!) Make a beautiful drawing to show your solution! How did you find it? Can you discover similar problems?

Have fun!