PFS 20-6 11/30/06 ________________

Iteration: Part Two

I. Squaring the Digits!

Start with any three- or four-digit number. Square each digit (multiply it by itself), and then add up the squares. Repeat! Look at the example begun here:

347	9 + 16 + 49 = 74
74	49 + 16 = 65
65	36 + 25 = 61

etc.

1) What eventually happens as you do this over and over? Do you reach a cycle? A fixed point? Do you return to your starting number? What will happen if you start with a different number? What if you start with a number with less than three digits or more than four?

2) Can you explain what's going on?!

II.. Square and Divide

In Squaring Digits there are thousands of numbers you can start with, so it's hard to see the whole picture. In this problem you can see the whole picture! Start with any number from 0 to 99. Square your number. Then divide by 100 and take the remainder. (This is the same as saying, take the two digits on the right.) Repeat! For example, start as follows:

84	84 x 84 = 7056
56	56 x 56 = 3136
36	36 x 36 = 1296

etc.

. In this problem there are 100 different starting numbers or seeds. Can you work in a group and find out what happens with every possible seed? (The job may be easier than you think! Will you really have to start it 100 times over?) If you wish, explore what will happen if you divide by a different number, such as 10, 11 or 12. (With 12, your starting numbers will be from 0 to 11.)

Have fun! Have fun! Have fun!

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Color Squares: Part II

Now that you have a complete set of 24 color squares, copy them neatly onto cardstock. Then cut out the squares *very* neatly. Now you'll have a set of 24 two-dimensional dominoes that can be tiled together in different ways. Try these:

1. (The warm-up.) Fit the 24 squares together to form a 4 x 6 rectangle. Edges that touch must be of the same color. The border colors don't matter.

2. (The challenge!) Fit the 24 squares together to form a 4 x 6 rectangle as above, but now the border must be all of one color! You'll be happy to know there are 12,261 different solutions to this puzzle, not counting rotations or reflections as different. Still, it is a little difficult to find even one!

3. Can you find more than one solution to #2?

4. Can you create a different tiling with your squares?

5. Can you invent a two-person game that can be played with your squares?

6. Have fun!

For more information: Martin Gardner's New Mathematical Diversions, 1966, pp 184--195.