PFS 20-3 10/16/06 ____________________

Adrian's Problem and Hip!

My friend Adrian remembered this problem from his student days in Australia and sent it our way. Start with 25 two-color counters on a five by five square board. All are red side up except the center one, which is orange side up. The goal is to get them all red side up. A move consists of flipping all the counters within a square of any size larger than one by one. (You can also use coins--heads and tails--etc.)

1) Try to solve Adrian's problem! Record your solution or how you worked on it. (It is possible!)

2) Try the same problem with the single orange starting in a different position. Can you make them all red? (Adrian says it's not possible now!) Can you find a proof or reason to explain why it's not possible?

3) Explore Adrian's problem on different size boards.

4) How many squares of all sizes are on a five by five board? An n x n board?

Hip is a game from Martin Gardner, the great math writer. It's played on a six by six square board. Players take turns placing down counters, a different color for each player. The first player to form a square with his counters loses the game. In this game, unlike Adrian's problem, we're thinking of squares fromed by four counters of the same color, and the square can be of any size, and can be rotated at different angles.

4) Play Hip! Can you develop a strategy to help you win? Do you think the first or second player has an advantage?

5) How many squares of all sizes and orientations are possible on a six by six board? Can you find a formula that will give that answer for different size boards?

6) Try a variation of Hip, invented by Mark Thompson, where each player places two counters down at a time, after their first single moves. What difference do you notice? Which game is more fun?

7) (For algebra students.) What do you notice about the slopes of the sides of the squares when the lines are neither vertical or horizontal?

8) Experiment with Hip on different size boards.

9) Have fun!

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A RE-RUN OF RACES

I. Race Results

Three people, Andy, Bobby and Chris, ran a race.

1. In how many different orders can they finish?

2. What if four people ran the race? Or two, or five? In how many orders could they finish? Is there a pattern or formula?

3. What if it's just three people, but we allow ties? (For example, Bobby first, Andy and Chris tied for second.) Then how many different orders are possible? What if other numbers are racing? Is there a formula now?

II. Tricky Track

Susie went to see a track meet between Harvard, Yale and Princeton, then reported about it at home. Her dad, who's a mathematician, asked her about the scoring.

"Well, Dad, each college entered one athlete in each event, and there were no ties in any event. You got a number for first, a lower number for second, and a still lower number for third place. The scoring was the same for each event."

"By 'number,' dear, I assume you mean 'natural number,'" said her dad.

"Sure, "natural number,' 'counting number,' 'positive integer,' whatever! You know, 1, 2, 3..."

"Hey, how did your Princeton friend do in the shot put?" asked her brother.

"Princeton won the shot-put," exclaimed Susie. "But Princeton didn't win the meet. The final score was Yale 22, Harvard 9, Princeton 9."

"How many events were there?" asked her dad.

"I don't know."

"Who won the mile run?" asked her sister.

"There was a mile run, but I didn't notice who won it."

Believe it or not, you now have enough information to answer the question: Which college won the mile run?!

Hints for Tricky Track:
Write down what you know and what you want to know.
Make a chart.
Use some trial and error.
Think about the problem globally: For example, not what did a team get for first place, but how many points were given out for each event.

Source for Tricky Track: Martin Gardner's New Mathematical Diversions