PFS 19-5 11/10/05 ___________________________

The Five Regular Polyhedra

Around the year 300 B.C.E. in Alexandria, Egypt, a man named Euclid published a math text written in Greek, called *The Elements.* It is the most influential math book ever written, and perhaps the most often translated and printed book ever. Euclid showed how math ideas could be linked together in a system, and how people could demonstrate truths to each other using proofs. His book has been studied throughout the world for over two thousand years, and studying Euclid has provided a foundation in reasoning for millions of people. Many great thinkers, such as the philosopher Spinoza and the physicist Newton, modeled their own great books on *The Elements.*

Book I of *The Elements* begins with the construction of the equilateral triangle, and Book XIII, the final book, ends with the construction of the five regular polyhedra, and the proof that there are only five.

So—what is a regular polyhedron? It's a solid whose faces are all regular polygons of one type, with the same number of faces meeting at each corner (or "vertex"). For example, a cube has six faces, all squares, and three squares meet at each vertex. (For the names of the other four, see the back of this page.)

1. Using the patterns you've been given, make a set of the five regular polyhedra. Remember to cut only around the border, and make all folds in the same direction. It may be easier to color before you fold. Remember to put your name on your polyhedra!

2. Enjoy their beauty.

3. Copy and complete the first chart on the back of this page. Can you notice a pattern? There is a rule relating the number of vertices, edges and faces!

4. Imagine a point in the center of each face of a cube. Now imagine connecting each of those points with its neighbors. What polyhedron would you get? (It's called the dual of the cube.) Do this same thought experiment for each polyhedron.

5. Pick a polyhedron, start at one vertex, and try to trace a path along all the edges, without tracing over any edge more than once. On which polyhedra is this possible?

6. Find the sum of the angles at each vertex of each solid. (For example, for the cube it's 90+90+90=270.) Is there a pattern?

7. Copy and complete charts 2 and 3 from the back of this page. What do you notice?

8. Which of these solids can "tile space?"

9. If you color your tetrahedron with a different color for each face, can you color others with the same four colors and tell them apart? Answer this question for the other polyhedra.

10. Explain why there are only five regular polyhedra.

11. Do some research about the regular polyhedra (sometimes called "Platonic solids") and their history.

12. Have fun!

Chart 1.

Name	Shape of each face	Vertices	Edges	Faces
Tetrahedron	triangle			4
Cube	square			6
Octahedron	triangle			8
Dodecahedron	pentagon			12
Icosahedron	triangle			20

Chart 2.

	Vertices	Edges	Faces	Cells	?
Point					
Segment					
Triangle					
Tetrahedron					
?					

Chart 3.

	Vertices	Edges	Faces	Cells	?
Point					
Segment					
Square					
Cube					
?					

PFS 19-5 Alternative 11/10/05 ______________________________

The Return of the Locker Problem!

At Euclid High School there is a long hallway with one hundred lockers in a row. One morning Student One arrives early and sees all one hundred lockers closed. She opens every one! Then Student Two arrives, and he closes lockers #2, # 4, # 6, etc. Now Student Three arrives, and she "changes the state" of lockers #3, #6, # 9, etc. In other words, if it's open, she closes it; and if it's closed, she opens it! Okay, here comes Student Four. He changes the state of every fourth locker, starting with #4. This pattern continues until Student One Hundred comes in and changes the state of locker #100.

Here's the question: at the end of this story, which lockers will be open?

There are many ways to approach this problem. Since 100 lockers is a lot to keep track of, you might want to start by working on the same problem with fewer lockers at first. Graph paper can help you keep track of the locker states. (I recommend "O" for open and "X" for closed.) Another way to think about this problem is to try to view it locally rather than globally. In other words, what will this story feel like for any one particular locker?

It's a great problem with a surprising answer!

Good luck!

Have fun!

If you get the answer, don't spoil other people's fun--let them work on it too!